Mathematics 3113-005 Name (please print) ____________________

Quiz 1 Form B

January 28, 2011

Instructions: Give concise answers, but clearly indicate your reasoning. Most of the problems have rather short answers, so if you find yourself involved in a lengthy calculation, it might be a good idea to move on and come back to that problem if you have time.

I. (3) Find infinitely many solutions of the differential equation $7y' = 5y$. Start by looking for a solution of the form $y = e^{rx}$, where r is a certain constant.

For $y = e^{rx}$, $y' = re^{rx}$. Putting these into the equation, we obtain $7re^{rx} = 5e^{rx}$, so $(7r-5)e^{rx} = 0$ and therefore $r = 5/7$. A solution is $y = e^{5x/7}$.

All functions of the form $y = Ce^{5x/7}$ are solutions.

II. (2) What basic types of mathematical objects (more basic than "integral") appear on the two sides of this well-known equation: $\int \cos(x)\,dx = \sin(x) + C$?

They are sets of functions.

III. (5) (a) Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{1}{x^2}$, $y(2) = 0$.

$y = \displaystyle\int \frac{1}{x^2}\,dx = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$. We want $0 = y(2) = -\dfrac{1}{2} + C$, so $C = \dfrac{1}{2}$ and therefore $y = \dfrac{1}{2} - \dfrac{1}{x}$.

(b) Apply the Existence and Uniqueness Theorem to this initial value problem (that is, verify that this initial value problem satisfies the hypotheses of the theorem). What does the theorem tell you about the solution you have found?

$\dfrac{\partial}{\partial y}\left(\dfrac{1}{x^2}\right) = 0$. Since $\dfrac{1}{x^2}$ and $\dfrac{\partial}{\partial y}\left(\dfrac{1}{x^2}\right)$ are continuous on any open rectangle containing the point $(2, 0)$, provided that the rectangle is to the right of the x-axis, our solution is unique.

IV. (2) Tell two well-known basic functions that are solutions of $y'' = -y$.

$\sin(x)$ and $\cos(x)$

V. (3) The separable differential equation $\dfrac{dy}{dx} = \dfrac{x}{y^2}$ can be writtten as $y^2\,dy = x\,dx$. Integrate both sides of this and solve for y to find the general solution.

Integrating gives $\dfrac{y^3}{3} = \dfrac{x^2}{2} + C$, so $y = (3x^2/2 + C)^{1/3}$.