Mathematics 3333-002 Name (please print) ____________________

Examination I Form A

February 19, 2010

Instructions: Give concise answers, but clearly indicate your reasoning. Most of the problems have rather short answers, so if you find yourself involved in a lengthy calculation, it might be a good idea to move on and come back to that problem if you have time.

I. (4) Given that $A^{-1} = \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix}$, find the inverse of AB.

II. (5) Use the row operation method to find the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ a & 0 \end{bmatrix}$ when $a \neq 0$ (the expression for the inverse will involve a in some way).

III. (4) Write a system of linear equations in x and y whose solutions, if any, will satisfy

$$\begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix} = x \begin{bmatrix} 1 & -1 \\ 4 & 0 \end{bmatrix} + y \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$$

but *do not solve the system* or try to find x and y.

IV. (4) Write any 3×3 matrix A with the property that

$$A \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}$$

V. (5) Find a condition on a and b which tells whether $\begin{bmatrix} a \\ b \end{bmatrix}$ is in the range of the matrix transformation

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2 & 2 & 1 \\ 4 & 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} .$$

VI. (9) Let A be an $m \times n$ matrix and consider a linear system $AX = B$, where X and B are vectors.

(a) What must be the dimensions of X and B, when they are regarded as matrices?

(b) Tell the *associated homogeneous system* of $AX = B$.

(c) Suppose that the system has at least one solution X_P. Show that if X_H is any solution of the associated homogeneous system, then $X_P + X_H$ is a solution of $AX = B$.

(d) Still assuming that the system has at least one solution X_P, show that *every* solution X of $AX = B$ is of the form $X_P + X_H$ for some solution X_H of the associated homogeneous system. [Hint: start by letting X be any solution. Then, what can you say about $X - X_P$?]

VII. (6) Find two elementary matrices E_1 and E_2 so that $E_2 E_1 A = I_3$, where $A = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$.

VIII. (8) Each of the following matrices is the augmented matrix of a system of linear equations, and is in row echelon form or reduced row echelon form. For each matrix, use back substitution to write a general expression for the solutions of the corresponding linear system.

1. $\begin{bmatrix} 0 & 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 1 & a & 0 & b \\ 0 & 1 & c & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (the answer will involve a, b, and c)

IX. (3) Give an example of two nonzero singular matrices A and B for which $A + B$ is nonsingular.

X. (5) Say you have $n \times n$ matrices A and B, and AB is nonsingular. Show that A must also be nonsingular. (That is, you can't *multiply* two singular matrices and get a nonsingular one.) (If you know about determinants, don't use them here. The solution must only use ideas that we have already studied.)