I. Let $A$ be the matrix
\[
\begin{bmatrix}
2 & 4 & -2 & 0 \\
1 & 3 & -1 & -3 \\
2 & 5 & -2 & -3
\end{bmatrix}.
\]
(a) Find a basis for the row space of $A$.
(b) Find a basis for the column space of $A$.

II. A certain $14 \times 10$ matrix $A$ has rank 8.
(a) What is the dimension of the solution space of the homogeneous system $AX = 0$?
(b) Can one solve the linear system $AX = B$ for all choices of $B$? Why or why not?

III. Let $V$ be a vector space with an inner product $(\_ , \_ )$.
(a) Let $w_0$ be a fixed vector in $V$. Show that the set of vectors orthogonal to $w_0$ is a subspace of $V$.
(b) Define what it means to say that a set of vectors $S = \{v_1, \ldots, v_n\}$ in $V$ is orthogonal.
(c) Define what it means to say that a set of vectors $S = \{v_1, \ldots, v_n\}$ in $V$ is orthonormal.

IV. By counting the number of inversions in the permutation 48253176 of eight elements, determine whether this permutation is even or odd.

V. Use the row operation method to calculate the determinant
\[
\begin{vmatrix}
3 & 4 & 2 \\
2 & 5 & 0 \\
6 & 2 & -1
\end{vmatrix}.
\]

VI. Let $A = \begin{bmatrix} t & 1 & 2 \\ -t & 1 & 1 \\ 0 & 2 & t \end{bmatrix}$.
(a) Calculate that $\text{det}(A) = 2t(t - 3)$ by using cofactor expansion of the determinant down the first column.
(b) Use the expression for $\text{det}(A)$ in part (a) (even if you did not carry out the calculation) to determine the values of $t$ for which $A$ is singular.

VII. As usual, let $P_3$ be the 4-dimensional vector space of polynomials of degree at most 3. Let $D: P_3 \to P_3$ be differentiation, a linear transformation. What is the kernel of $D$? What is the range of $D$? (Remark: there is no need to use a matrix representation. Just think about what polynomials are in the kernel and range.)
VIII. Let \( L : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) be the linear transformation defined by
\[
L \left( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} 2x - z \\ 2x - y \\ x + 2y - z \end{bmatrix},
\]

(a) Find the \textit{standard} matrix representation for \( L \), that is, find the matrix \( A \) so that \( AX = L(X) \) for every \( X \) in \( \mathbb{R}^3 \).

(b) Consider the ordered basis \( S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right\} \) for \( \mathbb{R}^3 \). By solving the system
\[
\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \lambda_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + \lambda_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}
\]
for \( \lambda_1, \lambda_2, \) and \( \lambda_3 \), obtain a formula of the form
\[
\begin{bmatrix} a \\ b \\ c \end{bmatrix}_S = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}
\]
to convert a vector \( v \) in \( \mathbb{R}^3 \) into its corresponding vector \( v_S \).

(c) Check that the formula that you found in part (b) really does convert the three vectors in the basis \( S \) to the standard basis vectors, that is, it should tell you that
\[
\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_S = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
\]
and similarly for the second and third vectors in \( S \). If it does not, go back and do part (b) correctly.

(d) Find the matrix representation of \( L \) \textit{with respect to the basis} \( S \) given in part (b). That is, find the matrix \( A \) so that \( Av_S = (L(v))_S \).