Math 6843 homework

15. (3/4) Let L and M be the standard longitude and meridian curves on the torus T. Check that with respect to the basis $\{L, M\}$, $(t_L)_\# = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ and $(t_M)_\# = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. Notice that $t_L^{-1} t_M$ is (isotopic to) the Anosov diffeomorphism studied in the previous problem.

16. (4/1) Let S be a hyperbolic surface. For simplicity, we will assume that S is closed, although it need only be of finite type.

    (a) Find a collection $\mathcal{C}$ of curves in S with the following properties: Any two intersect in 0 or 1 point, and the closure of each complementary region is a disk. The curves in the collection may be assumed to be geodesics (don't worry about the details of that).

    (b) Let j be an isometry of S (i. e. an isometry from S to S) that preserves each C in $\mathcal{C}$, and preserves the direction on each C. Show that j is the identity on each C.

    (c) Show that if j is an isometry as in (b) and is orientation-preserving, then j preserves each complementary region, and deduce that j is the identity.

    (d) Let j be an isometry of S. Prove that if j is isotopic to the identity map of S, then j equals the identity map.

    (e) Let j and k be isometries of S. Prove that if j is isotopic to k, then $j = k$.

    (f) Prove that the group $\mathrm{Isom}(S)$ of isometries of S is finite. [It suffices to show that the group $\mathrm{Isom}_+(S)$ of orientation-preserving isometries is finite. Let M be the maximum length of a curve in $\mathcal{C}$. Let S be the finite set of *oriented* geodesic curves in S of length $\leq M$ (so each geodesic curve appears twice in S, once with each orientation). Observe there is a homomorphism from $\mathrm{Isom}_+(S)$ into the permutation group on S. Show that the kernel is trivial.]

17. (4/8) Download and print out the document `mosher.pdf`, which has a link on our course web page. Spend some time reading it.