11. (2/21) Let \( \alpha \) and \( \beta \) be two curves on \( F = F_{2,0} \) which intersect in two points with the same orientations. Let \( G \) be a manifold regular neighborhood of \( \alpha \cup \beta \), like the neighborhoods considered in the proof that \( d(\alpha, \beta) \leq 2 \log_2(i(\alpha, \beta)) + 2 \), so that \( G \) deformation retracts to \( \alpha \cup \beta \).

1. Use the classification of surfaces to verify that \( G \) must be a twice-punctured torus \( F_{1,2} \). Hint: Show that \( G \) has \( \chi(G) = -2, g \geq 1, \) and \( b \geq 1 \).

2. Since \( G \) is a twice-punctured torus, \( S - G \) is either an annulus or the union of a once-punctured torus and a disk. On a standard picture of \( F_{2,0} \), try to draw an example of each type.

12. (2/21) For a (compact, connected, orientable) surface \( S \) with nonempty boundary, a pair of curves \( \alpha \) and \( \beta \) is said to fill \( S \) if every nontrivial curve in \( S \) must meet \( \alpha \cup \beta \). Equivalently, every component of \( S - (\alpha \cup \beta) \) is either an open disk or a half-open annulus \((0, 1) \times S^1\) that contains a boundary circle of \( S \). Try to draw two curves that fill the twice-punctured torus \( F_{1,2} \).

13. (2/28) Let \( A \) be a \( 2 \times 2 \) matrix with complex entries.

1. Check that the characteristic polynomial of \( A \) is \( \lambda^2 - \text{tr}(A)\lambda + \det(A) \).

2. Use the Cayley-Hamilton Theorem to deduce that if \( A \in \text{SL}(2, \mathbb{R}) \) and \( \text{tr}(A) = -1 \), then \( A^3 - I = 0 \), and hence \( A \) has order 3. Obtain similar results when \( \text{tr}(A) = 0 \) and \( \text{tr}(A) = 1 \).

3. Show that if \( A \in \text{SL}(2, \mathbb{R}) \) and \( |\text{tr}(A)| > 2 \), then \( A \) has two real eigenvalues which are reciprocals, one of which has absolute value greater than 1. Deduce that \( A \) has infinite order.

14. (2/28) Following the program we used with \[
\begin{bmatrix}
4 & 3 \\
5 & 4
\end{bmatrix}
\]
analyze the action of \[
\begin{bmatrix}
2 & 1 \\
1 & 1
\end{bmatrix}
\]
on the torus. That is:

1. Calculate the eigenvalues \( \{\lambda, 1/\lambda\} \) and associated length 1 eigenvectors \( v_\lambda \) and \( v_{1/\lambda} \).

2. Express the standard basis \( \{e_1, e_2\} \) (that correspond to the curves we call \( L \) and \( M \) on the torus) in terms of the basis \( \{v_\lambda, v_{1/\lambda}\} \). Use this to redraw the standard fundamental domain (spanned by \( e_1 \) and \( e_2 \)) using the basis \( \{v_\lambda, v_{1/\lambda}\} \).

3. Examine the stable and unstable foliations, first from the viewpoint of the basis \( \{v_\lambda, v_{1/\lambda}\} \). Find their slopes with respect to the standard basis, and try to imagine them on the torus and how the element \[
\begin{bmatrix}
2 & 1 \\
1 & 1
\end{bmatrix}
\]
acts on the torus.