Math 6843 homework

7. (1/31) Let F be a boundary component of a manifold M and let $H\colon M \to N$ be a homeomorphism, taking F to the boundary circle G of N. Let k be a homeomorphism from $F \to G$, and suppose that $H|_F$ is isotopic to k. Show that there is an isotopy from H to a homeomorphism H' such that $H' = H$ outside a small neighborhood of F and $H'|_F = k$. [Use the fact that F has a collar neighborhood $C = F \times I \subset M$ (which is "small", since a given one can always be replaced by $F \times [0, \epsilon]$) with $F = F \times \{0\}$. Each level H_t of the isotopy from H to H' will be H on $\overline{M - C}$, and $H_t|_F$ will be the given isotopy from $H|_F$ to k.]

8. (2/5) [This is nothing important, in fact, I've never seen it in a book— so please don't spend excessive time on it. I just thought it was fun to play with the algebra of connected sum.] Let $\mathcal{S}$ be the monoid of all compact, connected surfaces (up to homeomorphism), with the operation of connected sum. An *ideal* in $\mathcal{S}$ is a nonempty subset $\mathcal{J}$ such that if $J \in \mathcal{J}$, then $S\#J \in \mathcal{J}$ for all $S \in \mathcal{S}$. For example, the ideal generated by a subset $\{J_1, \ldots, J_n\}$ is the set of all $S\#J_i$ for $1 \le i \le n$ and $S \in \mathcal{S}$.

   1. What is the ideal generated by the disk?
   2. What is the ideal generated by the projective plane?

   For the remaining parts, this might be good preparation: Think about the orientable surfaces in the ideal generated by $F_{2,3}$. What do they look like if you identify $F_{g,b}$ with the point (g, b) in the plane?

   3. Show that the ideal generated by $F_{1,2}$ and $F_{2,1}$ cannot be generated by a single element. (I suppose you could say that $\mathcal{S}$ is not a principal ideal monoid.)
   4. Find a simple criterion that characterizes the ideals $\mathcal{J}$ for which $\mathcal{S} - \mathcal{J}$ is finite.
   5. Show that every ideal in $\mathcal{S}$ is finitely generated (I suppose you could say that $\mathcal{S}$ is a Noetherian monoid).

9. (2/5) Let $S = F_{g,b}$ with $\chi(F_{g,b}) = 2 - 2g - b < 0$, and let $\{C_1, C_2, \ldots, C_n\}$ be a pants decomposition of S. Show that $n = 3g + b - 3$ and that there are exactly $2g + b - 2$ pairs of pants in the decomposition.

10. (2/12) Hyperbolic space can also be modeled on the upper half plane $\{(x, y) \mid y > 0\}$; the geodesics with an endpoint at ∞ are vertical lines and the others are semicircles meeting the real line perpendicularly. Draw (some of) the Farey diagram in the upper half plane.