7. (1/31) Let $F$ be a boundary component of a manifold $M$ and let $H : M \to N$ be a homeomorphism, taking $F$ to the boundary circle $G$ of $N$. Let $k$ be a homeomorphism from $F \to G$, and suppose that $H|_F$ is isotopic to $k$. Show that there is an isotopy from $H$ to a homeomorphism $H' : M \to N$ taking $F$ to the boundary circle $G$ of $N$. Each level $H_t$ of the isotopy from $H$ to $H'$ will be $H$ on $M - C$, and $H_t|_F$ will be the given isotopy from $H|_F$ to $k$.

8. (2/5) [This is nothing important, in fact, I’ve never seen it in a book—so please don’t spend excessive time on it. I just thought it was fun to play with the algebra of connected sum.] Let $S$ be the monoid of all compact, connected surfaces (up to homeomorphism), with the operation of connected sum. An ideal in $S$ is a nonempty subset $J$ such that if $J \in J$, then $S \# J \in J$ for all $S \in S$. For example, the ideal generated by a subset $\{J_1, \ldots, J_n\}$ is the set of all $S \# J_i$ for $1 \leq i \leq n$ and $S \in S$.

1. What is the ideal generated by the disk?
2. What is the ideal generated by the projective plane?

For the remaining parts, this might be good preparation: Think about the orientable surfaces in the ideal generated by $F_{2,3}$. What do they look like if you identify $F_{g,b}$ with the point $(g, b)$ in the plane?

3. Show that the ideal generated by $F_{1,2}$ and $F_{2,1}$ cannot be generated by a single element. (I suppose you could say that $S$ is not a principal ideal monoid.)
4. Find a simple criterion that characterizes the ideals $J$ for which $S - J$ is finite.
5. Show that every ideal in $S$ is finitely generated (I suppose you could say that $S$ is a Noetherian monoid).

9. (2/5) Let $S = F_{g,b}$ with $\chi(F_{g,b}) = 2 - 2g - b < 0$, and let $\{C_1, C_2, \ldots, C_n\}$ be a pants decomposition of $S$. Show that $n = 3g + b - 3$ and that there are exactly $2g + b - 2$ pairs of pants in the decomposition.

10. (2/12) Hyperbolic space can also be modeled on the upper half plane $\{(x, y) \mid y > 0\}$; the geodesics with an endpoint at $\infty$ are vertical lines and the others are semicircles meeting the real line perpendicularly. Draw (some of) the Farey diagram in the upper half plane.