I. Let $a$ be a positive number and let $T$ be the triangle in the $xy$-plane bounded by $x = 0$, $y = 0$, and $x + y = a$. Find the centroid $(\overline{x}, \overline{y})$ of $T$ (i.e., the center of mass, assuming that $\rho = 1$) of $T$. You may take it as obvious that the centroid lies on the line $y = x$, so it is only necessary to calculate one of $\overline{x}$ or $\overline{y}$.

II. Let $E$ be the region in the first octant bounded by the surfaces $x^2 + y^2 = 1$, $z = 1$, and $y + z = 1$ (so $z = 1$ forms the top of the solid). Sketch the region, and supply limits of integration, in cylindrical coordinates, for the integral $\iiint_E f(x, y, z) \, dV$.

III. Evaluate $\int_0^1 \int_0^3 e^{x^2} \, dx \, dy$.

IV. Find the surface area of the part of the paraboloid $y = 1 - x^2 - z^2$ that has $y \geq 0$.

V. Find the $z$-coordinate $\overline{z}$ of the center of mass of the portion of the region $E$ in the first octant that lies inside the sphere $x^2 + y^2 + z^2 = 4$, assuming that the density is proportional to the distance from the origin.

VI. Let $S$ be the sphere of radius $a$ with center at the origin.

(a) The differential of surface area on $S$ can be expressed in terms of $d\phi$ and $d\theta$. Using the picture shown to the right, explain why $dS$ appears to be $a^2 \sin(\phi) \, d\phi \, d\theta$.

(b) Using the expression $dS = a^2 \sin(\phi) \, d\phi \, d\theta$, use a double integral in the variables $\phi$ and $\theta$ to calculate that the area of $S$ is $4\pi a^2$.

VII. Let $x = e^u \sin(t)$, $y = e^u \cos(t)$, and $z = f(x, y)$.

1. Calculate $\frac{\partial x}{\partial t}$ and $\frac{\partial y}{\partial t}$.

2. Calculate $\frac{\partial z}{\partial t}$ and express it purely in terms of $x$, $y$, $\frac{\partial z}{\partial x}$, and $\frac{\partial z}{\partial y}$.

3. Calculate $\frac{\partial}{\partial t} \left( \frac{\partial z}{\partial x} x \right)$ and express it purely in terms of $x$ and $y$ and partial derivatives of $z$. 
VIII. The figure to the right shows a change-of-coordinate function $\psi$, of the form $\psi(u,v) = (x(u,v), y(u,v))$.

(a) In the $xy$-coordinate system, sketch a possibility for what the vectors $\vec{r}_u = \frac{\partial x}{\partial u} \vec{i} + \frac{\partial y}{\partial u} \vec{j}$ and $\vec{r}_v = \frac{\partial x}{\partial v} \vec{i} + \frac{\partial y}{\partial v} \vec{j}$ might look like.

(b) Calculate the length of their cross product, and use it to write the relation between $dxdy$ and $dudv$.

IX. Consider the change-of-coordinate function $x = 3u$, $y = 2v$.

(a) Calculate the Jacobian of this change of coordinates.

(b) Find the curve in the $uv$-plane that corresponds to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

(c) Use this change of coordinates to find the area inside the ellipse, by calculating an integral in the $uv$-plane.