Mathematics 2443-006H Name (please print) ______________________

Examination I

February 14, 2008

Instructions: Give brief, clear answers.

I. (9) Calculate each of the following.

(a) The directional derivative of $\ln(x^2 + y^2)$ at the point $(2, 1)$ in the direction toward $(-1, 2)$.

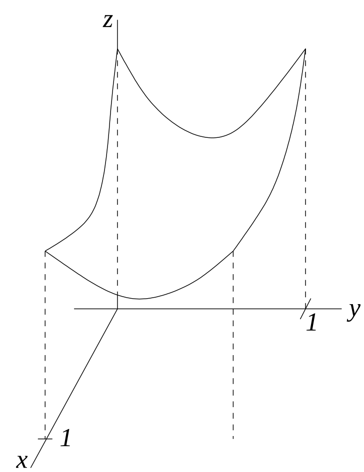
(b) The maximum rate of change of $qe^{-p} - pe^{-q}$ at $(p, q) = (0, 0)$, and the direction in which it occurs.

(c) An equation for the tangent plane to the level surface of $\sqrt{x^2 + y^2 + z^2}$ at the point $(1, 2, -2)$.

II. (6) Let $f(x, y) = xy - x + 2y$, and let D be the closed triangular region with vertices $(4, 0)$, $(0, 4)$, and $(0, 0)$. Find the maximum and minimum values of f on the domain D, and where they occur.

III. (5) Calculate the differential of the function $\sqrt{x^2 + y^2}$. Use it to calculate the linear part of the change of $\sqrt{x^2 + y^2}$ going from $(x, y) = (1, 1)$ to $(x, y) = (3, 2)$.

IV. (5) In an xy-coordinate system, make a reasonable sketch of the gradient of the function whose graph is shown at the right.



V. (4) Partition the domain $D = [0, 10] \times [0, 4]$ into six rectangles, using the partition $\{0, 2, 6, 10\}$ in the x-direction and $\{0, 2, 4\}$ in the y-direction. Using the midpoints as sample points, calculate the Riemann sum of the function $x - 2y$ for this partition.

VI. (7) Let $x = e^u \sin(t)$, $y = e^u \cos(t)$, and $z = f(x, y)$.

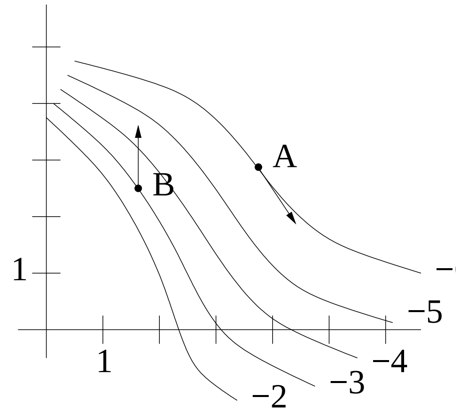
1. Calculate $\dfrac{\partial x}{\partial t}$ and $\dfrac{\partial y}{\partial t}$.
2. Calculate $\dfrac{\partial z}{\partial t}$ and express it purely in terms of x, y, $\dfrac{\partial z}{\partial x}$, and $\dfrac{\partial z}{\partial y}$.
3. Calculate $\dfrac{\partial}{\partial t}\left(\dfrac{\partial z}{\partial x} x\right)$ and express it purely in terms of x and y and partial derivatives of z.

VII. (4) Use implicit differentiation to calculate $\dfrac{\partial R}{\partial R_2}$ if

$$\frac{1}{\sin(R)} = \frac{1}{\sin(R_1 R_2)} + \frac{1}{\sin(R_1 R_3)} .$$

VIII. (6) In the xy-coordinate system to the right, the level curves $f(x, y) = c$ of a function are shown for $c = -2, -3, -4, -5$, and -6, along with two points A and B, and a unit vector at each of the points A and B.

1. Sketch reasonable possibilities for ∇f at the points A and B.
2. Make a reasonable guess of the rate of change of f at A in the direction of the vector shown there.
3. Make a reasonable guess of the rate of change of f at B in the direction of the vector shown there.



IX. (6) A unit vector $\vec{u}$ in 3-dimensional space can be written as $a\,\vec{\imath} + b\,\vec{\jmath} + c\,\vec{k}$ where a, b, and c are numbers satisfying $a^2 + b^2 + c^2 = 1$. Let $f(x, y, z)$ be a function on xyz-space.

(i) Write parametric equations for the straight line through the point (x_0, y_0, z_0) with direction vector $\vec{u} = a\,\vec{\imath} + b\,\vec{\jmath} + c\,\vec{k}$. (That is, find functions $x(t)$, $y(t)$, and $z(t)$ so that $x = x(t)$, $y = y(t)$, and $z = z(t)$ are parametric equations for this line.)

(ii) Put your explicit functions $x(t)$, $y(t)$, and $z(t)$ into the expression $f(x(t), y(t), z(t)))$ to find an expression for the values of f along the straight line. Use the Chain Rule to calculate $\frac{d}{dt}(f(x(t), y(t), z(t)))$.

(iii) Find the value of your expression for $\frac{d}{dt}(f(x(t), y(t), z(t)))$ when $t = 0$ and verify that it equals $\nabla f(x_0, y_0, z_0) \cdot \vec{u}$.