Mathematics 2423-001H Name (please print) _____

Final Examination

May 8, 2007

I. (8) Consider the portion of the graph $y = e^{-x}$ between $x = 0$ and $x = 1$. For each of the following, write an integral whose value is the specified quantity for this portion of the graph, but *do not* attempt to evaluate the integrals.

1. The length of this portion of the graph.
2. The surface area obtained when it is rotated about the x-axis.
3. The surface area obtained when it is rotated about the line $y = -1$.
4. The surface area obtained when it is rotated about the y-axis.

II. (6) Simpson's Rule states that $\displaystyle\int_a^b f(x)\,dx \approx \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 4y_{n-1} + y_n)$, with error of magnitude at most $\dfrac{K(b-a)}{180}h^4$, where $|f^{(4)}(x)| \le K$ for $a \le x \le b$. Use Simpson's rule with $n = 4$ to approximate $\displaystyle\int_{-1}^{1} x^4\,dx$, and give a bound for the error. Leave both answers as fractions, not decimals.

III. (6) Let C be the portion of the unit circle that lies in the first quadrant.

(i) Write the standard equation for C of the form $y = f(x)$, $0 \le x \le 1$, and calculate that $ds = \dfrac{1}{\sqrt{1-x^2}}\,dx$.

(ii) Integrate this to find the length of C. If the integral is improper, show the details of how you handle it.

IV. (3) Verify that $y = a\sinh(x) + b\cosh(x)$ is a solution to the differential equation $y'' = y$.

V. (6) State the Fundamental Theorem of Calculus (both parts, of course).

VI. (4) Calculate a Riemann sum for the function $f(x) = x^2$ on the interval $[0, 6]$, using the partition with $x_1 = 1$, $x_2 = 2$, and $x_3 = 4$, and using midpoints as the sample points.

VII. (8) For each of the following rational functions, write out the precise *form* of the partial fraction decomposition. *Do not* solve for unknown values of the coefficients.

1. $\dfrac{x^5 - x^2}{(x^3 + x)^3}$
2. $\dfrac{1}{(x^2 + x + 1)(x^2 + x - 1)}$

VIII. (6) Use l'Hôpital's rule to evaluate the following limits.

1. $\displaystyle\lim_{x \to 0^+} \sin(x)\,\ln(x)$
2. $\displaystyle\lim_{x \to 0} x^x$

IX. Evaluate the following integrals:
(20)

1. $\displaystyle\int \frac{\log_{10}(x)}{x}\,dx$
2. $\displaystyle\int \frac{\cosh(x)}{\cosh^2(x)-1}\,dx$
3. $\displaystyle\int_1^{\ln(5)} x^2\,e^x\,dx$
4. $\displaystyle\int \frac{1}{x^2+x+1}\,dx$
5. $\displaystyle\int x\,\cos^2(x)\,dx$

X. Find the domain and range of the function $f(x) = \ln(\tan^{-1}(x))$.
(4)

XI. Consider the function $y = e^{-x}$.
(8)

1. Calculate ds.
2. Write an improper integral whose value is the surface area produced when the graph of the function $y = e^{-x}$, $0 \le x < \infty$, is rotated about the x-axis.
3. Evaluate the integral, using the substitution $u = e^{-x}$ and the integration formula $\displaystyle\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln(u+\sqrt{a^2+u^2}) + C$.