Math 2423 homework

38. (4/24) Suppose that \( f(x) \) is a function whose third derivative \( f^{(3)}(x) \) exists and is continuous. Define \( E_2(h) \) by the formula
\[
f(a + h) = f(a) + f'(a)h + \frac{1}{2!}f''(a)h^2 + E_2(h).
\]

1. Use integration by parts to calculate that
\[
E_2(h) = \int_0^h \frac{1}{2!} (h - t)^2 f^{(3)}(a + t) \, dt.
\]

2. Let \( m \) be the minimum and \( M \) the maximum of \( f^{(3)} \) on the interval \([a, a + h]\).
Show that \( \frac{1}{3!} h^3 m \leq E_2(h) \leq \frac{1}{3!} h^3 M \).

3. Use the Intermediate Value Theorem to show that there exists \( c \) in \([a, a + h]\) so that
\[
E_2(h) = \frac{1}{3!} f^{(3)}(c) h^3.
\]

39. (4/24) 8.3 as many as needed from # 1-32, including at least 4-6, 21-24, 30-32

40. (4/24) 8.4 as many as needed from # 1-50, including at least 1, 5, 6, 23, 24, 29, 34, 38, 43, 44, 45, 49

41. (4/24) 8.6 # 17, 19, 22, 30, 31

42. 8.7 # 21(c) \((S_n \text{ only})\), 22, 46 (use the error formula)

43. 8.8 # 1, 2, as many as needed from 5-40 and 49-54

44. 8.8 # 71

45. 9.1 # 5, 7, 10-13

46. 9.2 # 5, 8, 9, 15, 16, 25