Mathematics 2513-001 Name (please print) ____________________

Examination I Form B

February 14, 2006

Instructions: Give brief, clear answers.

I. Using step-by-step logic, simplify the following expression: $\neg(\neg Q \wedge \forall z, (\ R(z) \Rightarrow P(z)\ )$
(3)

II. Write the following as an implication: "$b^2 \geq 2$ for at most one b".
(2)

III. Use a truth table to verify the tautology $\big(\neg Y \Rightarrow (X \wedge \neg X)\big) \Rightarrow Y$.
(4)

IV. Write each of the following as either $A \Rightarrow B$ or $B \Rightarrow A$:
(3) (i) whenever A, B
(ii) A is necessary for B
(iii) A, when B

V. Write the following statement in logical notation (and simplified so that it does not involve the negation
(3) symbol $\neg$) using the universal set $\mathcal{U} = \mathbb{Z}$: There is a positive integer that is not the sum of the squares of three integers.

VI. Give the general form of a proof by contradiction. That is, if the statement to be proven is P, give the main
(4) steps in the logical structure of the proof. Briefly explain why the argument proves the original assertion.

VII. In this problem, you may take as known the fact that $\sqrt{2}$ is irrational.
(9) (a) Prove that the difference of two rational numbers must be rational (that is, that if x and y are rational, then $x - y$ is rational).
(b) Prove that the sum of a rational number and an irrational number must be irrational.
(c) Give a counterexample to: The difference of two irrational numbers must be irrational.

VIII. Let $M(p, m)$ be "Person p has seen the movie m." Write each of the following statements in logical
(5) notation, putting in all necessary quantifiers using the sets $\mathcal{P}$ of all people and $\mathcal{M}$ of all movies. If your answer involves a negation, simplify as much as possible.
(a) Jeff has never seen a movie.
(b) Jack has seen every movie.
(c) Ellen has seen every movie that Max has seen.
(d) Everyone has seen at least one movie.
(e) Between the two of them, Jenny and Tom have seen every movie.

IX. Assuming that the universal set is $\mathcal{U} = \mathbb{R}$, prove the statement $\forall x, \exists y, x < y$.
(3)

X. Assuming that the universal set is $\mathcal{U} = \mathbb{R}$, prove (if the statement is true) or disprove (if the statement is
(8) false) each of the following statements.

1. $\forall x, (x > 1 \Rightarrow x > 0)$
2. $\exists x, (x > 1 \Rightarrow x > 0)$
3. $\forall x, (x > 0 \Rightarrow x > 1)$
4. $\exists x, (x > 0 \Rightarrow x > 1)$

XI. This problem concerns the following statement about integers: "If $3n + 6$ is even, then n is even."
(6) (a) Prove the statement by arguing the contrapositive.
(b) Prove the statement using proof by contradiction.