16. (3/9) Give formal proofs that the following functions are onto:

(i) \( f : \mathbb{R} \to \mathbb{R} \) defined by \( f(x) = 7x - 4 \).
(ii) \( f : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}_{\geq 0} \) defined by \( f(x, y, z) = x^2 + y^2 + z^2 \).

17. (3/9) Give formal proofs that the following functions are not onto:

(i) \( f : \mathbb{R} - \{n\pi | n \in \mathbb{Z}\} \to \mathbb{R} \) defined by \( f(x) = \csc(x) \).
(ii) \( f : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R} \) defined by \( f(x, y, z) = x^2 + y^2 + z^2 \).

18. (3/9) Give formal proofs that the following functions are injective:

(i) \( f : \mathbb{R} \to \mathbb{R} \) defined by \( f(x) = 7x - 4 \).
(ii) \( f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R} \) defined by \( f(x, y) = (2x + y, x + y) \).

19. (3/9) Give formal proofs that the following functions are not injective:

(i) \( f : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \) defined by \( f(m, n) = mn \).
(ii) \( f : \mathbb{R} \to \mathbb{R} \) defined by \( f(x) = x^2 - 3x + 4 \).

20. (3/9) 1.8 # 16

21. (3/30) 1.8 # 17, 25, 31

22. (3/30) 1.8 # 26 (The answer is “yes”— assume that \( f \) and \( f \circ g \) are injective and deduce that \( g \) is injective. Did you even need the assumption that \( f \) was injective?), 27 (The answer is “no”. There are many examples of \( f \) and \( g \) for which the assertion is false— one such example can be given using squaring function with various domains and codomains)

23. (3/30) 1.8 # 34 (“inverse image” is what I called “preimage of a set”), 35 (the “floor” function \( \lfloor \cdot \rfloor : \mathbb{R} \to \mathbb{Z} \) is defined by putting \( \lfloor x \rfloor \) equal to the largest integer that is less than or equal to \( x \)), 36 (it is convenient to use the observation that \( x \in f^{-1}(B) \iff f(x) \in B \))

24. (3/30) 2.4 # 2, 4, 6, 7, 8 (if you want, use a Google search to find a list of primes), 12, 13

25. (4/13) Know Euclid’s proof that there are infinitely many primes.

26. (4/13) Prove that \( a|n \iff a| - n \).

27. (4/13) 2.4 #16, 17