Math 2513 homework

16. (3/9) Give formal proofs that the following functions are onto:
    (i) $f\colon \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 7x - 4$.
    (ii) $f\colon \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}_{\geq 0}$ defined by $f(x, y, z) = x^2 + y^2 + z^2$.

17. (3/9) Give formal proofs that the following functions are not onto:
    (i) $f\colon \mathbb{R} - \{n\pi \mid n \in \mathbb{Z}\} \to \mathbb{R}$ defined by $f(x) = \csc(x)$.
    (ii) $f\colon \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined by $f(x, y, z) = x^2 + y^2 + z^2$.

18. (3/9) Give formal proofs that the following functions are injective:
    (i) $f\colon \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 7x - 4$.
    (ii) $f\colon \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ defined by $f(x, y) = (2x + y, x + y)$.

19. (3/9) Give formal proofs that the following functions are not injective:
    (i) $f\colon \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ defined by $f(m, n) = mn$.
    (ii) $f\colon \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 - 3x + 4$.

20. (3/9) 1.8 # 16

21. (3/30) 1.8 # 17, 25, 31

22. (3/30) 1.8 # 26 (The answer is "yes"— assume that f and $f \circ g$ are injective and deduce that g is injective. Did you even need the assumption that f was injective?), 27 (The answer is "no". There are many examples of f and g for which the assertion is false— one such example can be given using squaring function with various domains and codomains)

23. (3/30) 1.8 # 34 ("inverse image" is what I called "preimage of a set"), 35 (the "floor" function $\lfloor\ \rfloor\colon \mathbb{R} \to \mathbb{Z}$ is defined by putting $\lfloor x \rfloor$ equal to the largest integer that is less than or equal to x), 36 (it is convenient to use the observation that $x \in f^{-1}(B) \Leftrightarrow f(x) \in B$)

24. (3/30) 2.4 # 2, 4, 6, 7, 8 (if you want, use a Google search to find a list of primes), 12, 13

25. (4/13) Know Euclid's proof that there are infinitely many primes.

26. (4/13) Prove that $a|n \Leftrightarrow a| - n$.

27. (4/13) 2.4 #16, 17