I. Let $\vec{F}$ be the vector field $\frac{-y}{x^2+y^2}\vec{i} + \frac{x}{x^2+y^2}\vec{j}$. Verify by calculation (direct or indirect) that $\int C \vec{F} \cdot d\vec{r}$ is not path-independent on the domain $\{(x,y) \mid (x,y) \neq (0,0)\}$.

II. (a) Evaluate the line integral $\int_C xy \, dx + x^2y \, dy$ directly, where $C$ is the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$.
(b) Evaluate it using Green’s Theorem.

III. The figure to the right shows a vector field $P\vec{i} + Q\vec{j}$ on a portion of the plane. Based on its appearance there:
(a) Explain geometrically why $\frac{\partial P}{\partial y}$ is positive.
(b) Explain geometrically why $\frac{\partial P}{\partial y}$ is negative.
(c) Explain geometrically why $\frac{\partial Q}{\partial x}$ is zero.
(d) Explain geometrically why $\frac{\partial Q}{\partial y}$ is positive.
(e) Determine whether $\text{div}(P\vec{i} + Q\vec{j})$ is positive or negative.
(f) Determine whether $\text{curl}(P\vec{i} + Q\vec{j}) \cdot \vec{k}$ is positive or negative.

IV. Let $\vec{F}$ be a vector field as $P\vec{i} + Q\vec{j} + R\vec{k}$, verify that $\text{div}(f\vec{F}) = f\text{div}(\vec{F}) + \vec{F} \cdot \nabla f$.

V. (a) Sketch the vector field $\frac{-y}{x^2+y^2}\vec{i} + \frac{x}{x^2+y^2}\vec{j}$.
(b) Explain the important phenomenon (related to Clairaut’s Theorem) that the vector field $\frac{-y}{x^2+y^2}\vec{i} + \frac{x}{x^2+y^2}\vec{j}$ illustrates.

VI. Let $D$ be a connected planar domain.
(a) Define what it means to say that $D$ is simply-connected (do better than “no holes”).
(b) State the theorem discussed in class that uses the hypothesis that $D$ is simply-connected.

VII. Suppose that $\vec{F} = P\vec{i} + Q\vec{j}$ is a vector field on the plane and let $C$ be the unit circle. Suppose that at points on $C$ (but not necessarily on the rest of $D$), $\vec{F}(x,y) = x\vec{i} + y\vec{j}$.
(a) Verify that on $C$, $\vec{F}$ equals the outward unit normal $\vec{n}$.
(b) Calculate $\iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \, dA$.
(c) Calculate $\iint_D \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \, dA$.

VIII. Calculate $\int_C (y\vec{i} - x\vec{j}) \cdot d\vec{r}$, where $C$ is the equilateral triangle that has one side equal to the straight line from $(1,1)$ to $(201,1)$ but does not lie completely in the first quadrant.