Mathematics 2443-006H — Name (please print) ______________________

Examination II

March 28, 2006

Instructions: Give brief, clear answers.

**I.** (5) Use a double integral in polar coordinates to calculate the area bounded by the circle $x^2 + y^2 = ax$.

**II.** (4) Rewrite the integral $\int_{-\sqrt{3}}^{\sqrt{3}} \int_{-\sqrt{3-x^2}}^{0} f(x,y)\,dy\,dx$ to integrate first with respect to $x$, then with respect to $y$.

**III.** (6) Consider the solid cylinder bounded by $x^2 + z^2 = 4$ and the planes $y = 0$ and $y = 1$, and let $E$ be the points in this solid cylinder that have $z \geq 0$. Suppose that the density of $E$ at a point $(x, y, z)$ equals twice the distance from $(x, y, z)$ to the $y$-axis. Calculate the mass of $E$.

**IV.** (5) Let $D$ be the unit disk in the $xy$-plane. Write an integral in polar coordinates to calculate the surface area of the portion of $z = e^{x^2+y^2}$ lying above $D$. Simplify the integrand, and supply limits of integration, but do not continue further in evaluation of the integral.

**V.** (4) Let $D$ be the unit disk in the $xy$-plane, and consider the function $f(x,y) = \dfrac{1}{e^{x^2+y^2}}$, whose values depend only on $r$. Obtain a partition of $D$ by cutting it into four quarters, each consisting of the intersection of $D$ with one of the four quadrants. Calculate the smallest and largest Riemann sums for $f$ for this partition.

**VI.** (8) Calculate the area inside the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ as follows.

(a) Let $S$ be the region bounded by the ellipse. Define $\phi$ from the $uv$-plane to the $xy$-plane by $\phi(u, v) = (au, bv)$. Determine the region $R$ in the $uv$-plane that corresponds to $S$ under $\phi$.

(b) Calculate the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{pmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial y}{\partial u} \\ \dfrac{\partial x}{\partial v} & \dfrac{\partial y}{\partial v} \end{pmatrix}$ and its determinant.

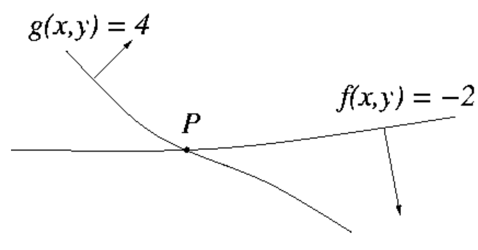
(c) Write a double integral over the domain $S$ whose value is the area, change it into $uv$-coordinates, and evaluate to find the area.

**VII.** (6) Consider the function $f(x,y) = x^4 + y^4 - 4xy + 2$ on the square $D = \{(x,y) \mid -2 \leq x \leq 2, -2 \leq y \leq 2$.

(a) Find all critical points of $f$ on this domain.

(b) Show that extreme values of $f$ on the right-hand vertical boundary side of $D$ can occur only at one of the three points $(2, \pm 2)$ or $(2, \sqrt[3]{2})$.

**VIII.** (4) The figure to the right shows level curves $f(x,y) = -2$ and $g(x,y) = 4$ of two functions $f$ and $g$ in the $xy$-plane, their intersection point $P$, and a gradient vector for each of the functions at a point on its level curve. Let $h$ be the function defined by $h(x,y) = f(x,y)\,g(x,y)$. There are four directions in which one can leave $P$ moving along one of the level curves. For each of them, label whether $h$ is increasing or decreasing as one moves away from $P$ in that direction. (Caution: Before answering, think carefully about the way that a product changes when one of the factors is negative.)




**IX.** (6) Use integration in spherical coordinates to find the volume of the region $E$ bounded by $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 + z^2 = 3$.

**X.** (6) Let $E$ be the upper hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$, and consider the integral $\iiint_E f(x,y,z)\,dV$.

(a) Rewrite the integral in cylindrical coordinates, supplying limits of integration appropriate for $E$.

(b) Rewrite the integral in spherical coordinates, supplying limits of integration appropriate for $E$.