35. (3/22) Recall that if $H$ is a subgroup of a group $G$, then $gHg^{-1}$ is the subgroup consisting of all elements $ghg^{-1}$ for $h \in H$, and recall that $H$ is called a normal subgroup if $gHg^{-1} = H$ for all $g \in G$.

1. Verify that every subgroup of an abelian group is normal.

2. Verify that the subgroup consisting of the powers of $\alpha$ is a normal subgroup of $D_n$.

3. Verify that the subgroup consisting of the powers of $\beta$ is a normal subgroup of $D_n$ if and only if $n \leq 2$.

4. Let $T$ be the subgroup of $\text{Isom}_+ (\mathbb{R}^2)$ consisting of all translations, that is, all elements of the form $T_v$. Verify that $T$ is isomorphic to $\mathbb{R}^2$, and is a normal subgroup of $\text{Isom}_+ (\mathbb{R}^2)$.

5. Let $R$ be the subgroup of $\text{Isom}_+ (\mathbb{R}^2)$ consisting of all rotations, that is all elements of the form $R_\alpha$. Verify that $R$ is isomorphic to $S^1$, and is not a normal subgroup of $\text{Isom}_+ (\mathbb{R}^2)$.

6. Find a subgroup of $\text{Isom}(\mathbb{R}^2)$ (not $\text{Isom}_+ (\mathbb{R}^2)$, as you will want to use the isometry $\tau(x, y) = (x, -y)$) that is isomorphic to $D_n$.

36. (3/22) Consider the quotient space of the standard 2-sphere $S^2$ in $\mathbb{R}^3$, obtained by identifying each $x$ with $-x$.

1. Show that the quotient space is homeomorphic to the real projective plane $P = \mathbb{RP}^2$, obtained from a Möbius band and a 2-disk by identifying their boundary circles.

2. Let $p: S^2 \to P$ be this quotient map. Show (a good picture should be enough) that each $x \in P$ has an open neighborhood $U$ for which $p^{-1}(U)$ consists of two copies of $U$, each mapped homeomorphically to $U$ by the restriction of $p$.

3. The previous condition implies that $p: S^2 \to P$ satisfies the unique path lifting and unique homotopy lifting theorems, just as with the map $\mathbb{R} \to S^1$ (no need to prove this, the argument is exactly the same). Use these to prove that $\pi_1(P) \cong C_2$ (the fact that $S^2$ is simply-connected, which we proved in class since we proved that $\pi_1(S^2) = \{1\}$, is needed in the argument).

37. (4/5) Let $X$ and $Y$ be spaces, and give the set of continuous functions $C(X, Y)$ the C-O topology. Show that if $Y$ is Hausdorff, then $C(X, Y)$ is Hausdorff.

38. (4/5) Let $X$ and $Y$ be spaces, and give the set of continuous functions $C(X, Y)$ the C-O topology. Prove that if $A$ is a subspace of $X$, then the function $C(X, Y) \to C(A, Y)$ determined by sending $f$ to $f|_A$ is continuous.