Math 5863 homework

51. (highly optional problem) Let G be the group $\mathrm{Isom}_+(\mathbb{R}^2)$ of orientation-preserving isometries of the plane $\mathbb{R}^2$. Let H be the subgroup of G consisting of translations by vectors of the form (m, n), where m and n are integers, and as usual let T be the subgroup consisting of all translations.

    1. Show that H is normal in T and T is normal in G, but that H is not normal in G.
    2. Let H be the subgroup of G consisting of translations by vectors of the form (m, n), where m and n are integers. Verify that $T \subseteq N(H)$.
    3. It is true that T has index 4 in $N(H)$. Find coset representatives for the four cosets. Hint: Remember that $R_\theta \circ T_v \circ R_{-\theta} = T_{R_\theta(v)}$. So if R_θ is in the normalizer of H, then R_θ must take integer vectors to integer vectors.