51. (highly optional problem) Let $G$ be the group $\text{Isom}_+(\mathbb{R}^2)$ of orientation-preserving isometries of the plane $\mathbb{R}^2$. Let $H$ be the subgroup of $G$ consisting of translations by vectors of the form $(m, n)$, where $m$ and $n$ are integers, and as usual let $T$ be the subgroup consisting of all translations.

1. Show that $H$ is normal in $T$ and $T$ is normal in $G$, but that $H$ is not normal in $G$.

2. Let $H$ be the subgroup of $G$ consisting of translations by vectors of the form $(m, n)$, where $m$ and $n$ are integers. Verify that $T \subseteq N(H)$.

3. It is true that $T$ has index 4 in $N(H)$. Find coset representatives for the four cosets. Hint: Remember that $R_\theta \circ T_v \circ R_\theta = T_{R_\theta(v)}$. So if $R_\theta$ is in the normalizer of $H$, then $R_\theta$ must take integer vectors to integer vectors.