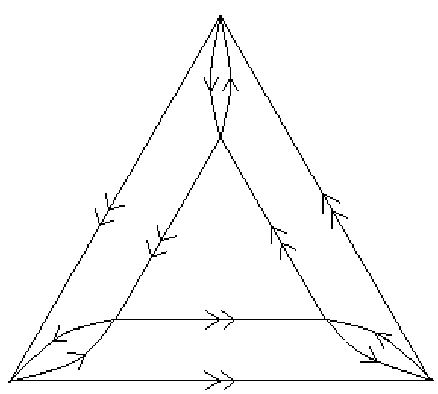
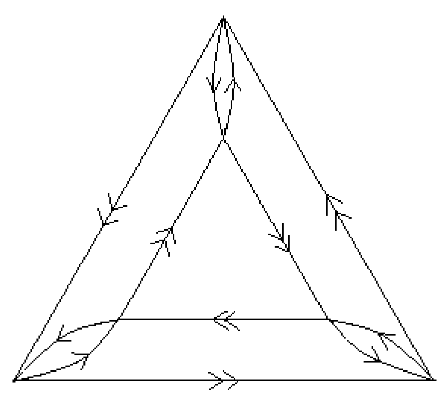
Math 5863 homework

49. Two 6-fold covering spaces of $S^1 \vee S^1$ are shown below, each with basepoint e which the top vertex of the outer triangle. Each covering space has six covering transformations. To see this, notice that e can only go to one of the six vertex points. By the uniqueness of lifts, two covering transformations that agree on e must be equal, and finally for each of the six vertex points there does indeed exist a covering transformation taking e to that vertex.

    1. Find a covering transformation of order six for the first covering space. This tells us that its group of covering transformations is C_6.
    2. Find the order of each covering transformation of the second covering space, in particular notice that it has no covering transformation of order six. This tells us that its group of covering transformations is the other group of order six, that is, $D_3 = \langle a, b \mid bab^{-1} = a^{-1}$. Choose one of the order-three covering transformations to be a, one of the order-two ones to be b, and verify that bab^{-1} equals a^{-1}.
    3. For each of the two spaces, choose a covering transformation b of order 2, and find the quotient space $E/\sim$, where $x \sim b(x)$ for every x (these will be two 3-fold covering spaces of $S^1 \vee S^1$.
    4. Find the group of covering transformations for each of the two 3-fold coverings of $S^1 \vee S^1$ from the previous part.





50. Let G be the dihedral group of order 8, in which $a^4 = b^2 = 1$ and $bab^{-1} = a^{-1}$. Let $H = \{1, b\}$, a cyclic subgroup of G of order 2.

    1. Show that H is not normal in G.
    2. Determine the normalizer $N(H)$.
    3. Determine $N(H)/H$.