1. (1/18) The Klein bottle $K$ can be constructed from two annuli $A_1$ and $A_2$ by identifying their boundaries in a certain way. For each of the three descriptions of $K$ discussed in class (two Möbius bands with boundaries identified, the square with certain identifications on its boundary, and $S^1 \times I$ with the two ends identified), make a drawing showing where $A_1$ and $A_2$ appear in $K$.

2. (1/18) Two surfaces $F_1$ and $F_2$ can be constructed as follows. Start with $S^1 \times I$, and remove the interior of a small disk $D$ from the interior of $S^1 \times I$. For $F_1$, identify each $(\theta, 0)$ with $(\theta, 1)$ and identify each point of $\partial D$ with its antipodal point (that is, if $\partial D$ is regarded as $S^1$, then $v$ is identified with $-v$). For $F_2$, identify each $(\theta, 0)$ with $(\bar{\theta}, 1)$ and identify each point of $\partial D$ with its antipodal point.

1. Make drawings illustrating each of $F_1$ and $F_2$. Notice that both are closed surfaces.
2. Find three disjoint Möbius bands imbedded in $F_1$.
3. Find three disjoint Möbius bands imbedded in $F_2$.

Actually, $F_1$ and $F_2$ are homeomorphic, although this may not be very easy to see.

3. (1/18) Let $M$ and $N$ be $n$-dimensional manifolds, and let $U$ be an open subset of $M$. Suppose that $f: U \to N$ is a continuous injection. Prove that $f$ takes open sets in $U$ to open sets in $N$.

4. (1/25) Prove that the relation $\simeq$ of being homotopic is an equivalence relation on the set of continuous maps from $X$ to $Y$.

5. (1/25) Let $X$ be a one-point space, $X = \{\ast\}$. Prove that the homotopy classes of continuous maps from $X$ to $Y$ correspond bijectively to the path components of $Y$.

6. (1/25) Suppose that $f_0, f_1: X \to Y$ are homotopic. Prove that if $g: Y \to Z$ is a continuous map, then $g \circ f_0 \simeq g \circ f_1$. Prove that if $k: Z \to X$ is a continuous map, then $f_0 \circ k \simeq f_1 \circ k$.

7. (1/25) Recall that the cone on $X$, $C(X)$, is the quotient space obtained by identifying the subspace $X \times \{1\}$ of $X \times I$ to a point. We identify $X$ with the subspace $X \times \{0\}$ of $C(X)$, by letting $x$ correspond to the point $[(x, 0)]$. Let $f: X \to Y$ be a continuous map. Prove that $f$ is homotopic to a constant map if and only if there exists a continuous map $g: C(X) \to Y$ for which $g|_X = f$. 