Mathematics 2423-001H Name (please print) ______________________

Examination II

March 24, 2005

I. (6) Let R be the region bounded by $y = \dfrac{1}{x^2+1}$, the x-axis, the y-axis, and the line $x = 2$. Calculate the volume produced when this region is rotated about the y-axis.

II. (6) Calculate the volume produced when the region bounded by the curve $y = e^x$, the x-axis, the y-axis, and the line $x = 1$ is rotated about the line $y = -c$, where c is a positive number.

III. (6) Define what it means to say that a function f is *one-to-one*. Find the smallest value of a for which the function defined by $f(x) = 3x^2 + 17x + 217$ is one-to-one on the interval $[a, \infty)$ (i. e. for $a \le x < \infty$).

IV. (7) Let $a > 1$. Explain why the derivative of the function a^x at $x = 0$ is $\lim_{h\to 0} \dfrac{a^h - 1}{h}$. Writing a_0 for the number $\lim_{h\to 0} \dfrac{a^h - 1}{h}$, show that the derivative of a^x is $a_0\, a^x$.

V. (6) Solve the following equations for x.

1. $2\ln(x) = \ln(2) + \ln(x+1)$.
2. $e^{ax} = Ce^{bx}$.

VI. (12) Calculate the following derivatives.

1. $\dfrac{dy}{dt}$ if $y = \ln\left(\sqrt[4]{\left(\dfrac{(2t+1)^3}{t^2-1}\right)^5}\right)$.
2. $\dfrac{dy}{dx}$ if $y = x^{1/x}$.
3. $\dfrac{d}{dx}\big(f^{-1}(x)\big)$ in terms of f'.

VII. (12) Calculate the following integrals.

1. $\displaystyle\int \frac{e^x + 1}{e^x}\, dx$.
2. $\displaystyle\int \frac{e^x}{e^x + 1}\, dx$.
3. $\displaystyle\int x\, 2^{x^2}\, dx$.
4. $\displaystyle\int_e^6 \frac{dx}{x\ln(x)}$.

VIII. (5) For $x > 1$, let $M(x)$ be the average value of the natural logarithm function on the interval from 1 to x. Write an expression for $M(x)$. Verify that $M(x) + (x-1)M'(x) = \ln(x)$.

IX. (5) How are the volume of an object and its average cross-sectional area related?