I. Let $R$ be the region bounded by $y = \frac{1}{x^2 + 1}$, the $x$-axis, the $y$-axis, and the line $x = 2$. Calculate the volume produced when this region is rotated about the $y$-axis.

II. Calculate the volume produced when the region bounded by the curve $y = e^x$, the $x$-axis, the $y$-axis, and the line $x = 1$ is rotated about the line $y = -c$, where $c$ is a positive number.

III. Define what it means to say that a function $f$ is one-to-one. Find the smallest value of $a$ for which the function defined by $f(x) = 3x^2 + 17x + 217$ is one-to-one on the interval $[a, \infty)$ (i.e. for $a \leq x < \infty$).

IV. Let $a > 1$. Explain why the derivative of the function $a^x$ at $x = 0$ is $\lim_{h \to 0} \frac{a^h - 1}{h}$. Writing $a_0$ for the number $\lim_{h \to 0} \frac{a^h - 1}{h}$, show that the derivative of $a^x$ is $a_0 a^x$.

V. Solve the following equations for $x$.

1. $2 \ln(x) = \ln(2) + \ln(x + 1)$.

2. $e^{ax} = Ce^{bx}$.

VI. Calculate the following derivatives.

1. $\frac{dy}{dt}$ if $y = \ln \left( \frac{\sqrt{(2t + 1)^3} - 5}{t^2 - 1} \right)$.

2. $\frac{dy}{dx}$ if $y = x^{1/x}$.

3. $\frac{d}{dx} \left( f^{-1}(x) \right)$ in terms of $f'$.

VII. Calculate the following integrals.

1. $\int \frac{e^x + 1}{e^x} \, dx$.

2. $\int \frac{e^x}{e^x + 1} \, dx$.

3. $\int x^2 e^x \, dx$.

4. $\int_0^1 \frac{dx}{x \ln(x)}$.

VIII. For $x > 1$, let $M(x)$ be the average value of the natural logarithm function on the interval from 1 to $x$. Write an expression for $M(x)$. Verify that $M(x) + (x - 1)M'(x) = \ln(x)$.

IX. How are the volume of an object and its average cross-sectional area related?