Mathematics 2423-001H Name (please print) ____________________

Examination I

February 15, 2005

I. (5) Use the telescoping sum $\sum_{k=1}^{n} k^2 - (k-1)^2$ to obtain the formula $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

II. (5) Calculate the Riemann sum for the following partition and function, using left-hand endpoints as the sample points x_i^*: the function is $f(x) = x^2/2$, the interval is $[1, 10]$, and the partition is $x_1 = 2$, $x_2 = 4$, and $x_3 = 9$.

III. (3) Give an explicit example of a partition of the interval $[0, 10]$ that has mesh π.

IV. (5) Let $f(x)$ be the function defined by $f(x) = 0$ for $0 \leq x < 5$ and $f(x) = 1$ for $5 \leq x \leq 10$. Consider the partition of $[0, 10]$ defined by $x_1 = 3$, $x_2 = 7$. By making two different choices of the points x_i^*, show that both of the numbers 3 and 7 are Riemann sums for this function and this partition of $[0, 10]$.

V. (3) Write the following limit as an integral, but do not try to calculate the integral: $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{6n} \tan\left(\frac{i\pi}{6n}\right)$.

VI. (6) State the Fundamental Theorem of Calculus (both parts, of course).

VII. (3) State the Mean Value Theorem for Integrals.

VIII. (6) Calculate the following derivatives: $\frac{d}{dx}\int_1^x \frac{\sin(t)}{t}\,dt$, $\frac{d}{dx}\int_1^{x^3} \frac{\sin(t)}{t}\,dt$, $\frac{d}{dx}\int_{x^2}^{x^3} \frac{\sin(t)}{t}\,dt$.

IX. (3) Verify that $\int (x^2-1)^{3/2}\,dx$ is **not** $\frac{2}{5}(x^2-1)^{5/2} + C$.

X. (9) Calculate the following indefinite integrals: $\int \left(w + \frac{1}{w}\right)^2 dw$, $\int \sqrt{\cot(x)}\csc^2(x)\,dx$, and $\int \frac{\cos(\pi/x)}{x^2}\,dx$.

XI. (4) Calculate $\int_0^{3\pi/2} |\cos(\theta)|\,d\theta$.

XII. (5) A differentiable function $f(x)$ satisfies $f(100) = 100$ and $f'(x) < \frac{1}{x}$ for all x. Show that $f(1000) < 109$.

XIII. (4) Use the substitution $u = \sin(\theta)$ (and the fact that $\cos^2(\theta) = 1 - \sin^2(\theta)$) to calculate that the following integral $\int_0^{\pi} \sin^5(\theta)\cos^7(\theta)\,d\theta$ equals 0.

XIV. (4) Simplify $x^2 - x^4 + x^6 - x^8 + x^{10} - \cdots + x^{202}$.