Math 2423 homework

1. (due 2/1) Use the telescoping sum $\sum_{k=1}^{n} k^4-(k-1)^4$ and the formulas that we established for $\sum_{k=1}^{n} k$ and $\sum_{k=1}^{n} k^2$ to obtain the formula $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$.

2. (2/1) Give a simple formula for $\sum_{k=0}^{n}(-1)^k x^k$ (the answer involves the expression $(-1)^n$).

3. (2/1) Let f be a function which is differentiable everywhere. For the error term $E(h)$ in $f(a+h) = f(a)+f'(a)h+E(h)$, use the Mean Value Theorem to obtain the estimate that for some c between a and $a+h$, $|E(h)| \leq |f''(c)|h^2$.

4. (2/1) Use the previous problem to show that $|\sin(x) - x| \leq x^2$ for all x.

5. (2/1) 5.1 # 1, 3

6. (2/1) Water is poured into a tank at a varying rate. Let $V(t)$ be the volume of water at time t. For a short time interval $[t_{i-1}, t_i]$ and a sample point t_i^* in $[t_{i-1}, t_i]$, explain what is approximated by $V'(t_i)\Delta t_i$. Use Riemann sums to explain intuitively why $V(b) - V(a) = \int_a^b V'(t)\,dt$.

7. (2/1) Let δ be a number with $0 < \delta < 1$. Construct a partition of $[0, 1]$ whose mesh is exactly δ (write it down precisely if you can, if not just explain it geometrically).

8. (2/1) 5.1 # 20, 21

9. (2/1) 5.2 # 17-20, 33-40

10. (2/8) 5.2 # 46-50, 67-69

11. (2/8) 5.3 # 10-17, 19-36 (as many as needed), 45-48, 53, 54

12. (2/15) 5.4 # 1-2, 5-14 (as many as needed), 17-40 (as many as needed), 43-44, 58, 60

13. (2/15) 5.5.# 1-32 (as many as needed), 37-54 (as many as needed), 57, 58, 61, 62, 64, 65

14. (3/1) 6.1 # 1-26 (as many as needed), 44-46

15. (3/1) 6.2 # 1-18 (as many as needed), 31-36 (as many as needed), 41-44, 48, 49, 62

16. (3/1) 6.3 # 3-26 (as many as needed), 29-32, 46