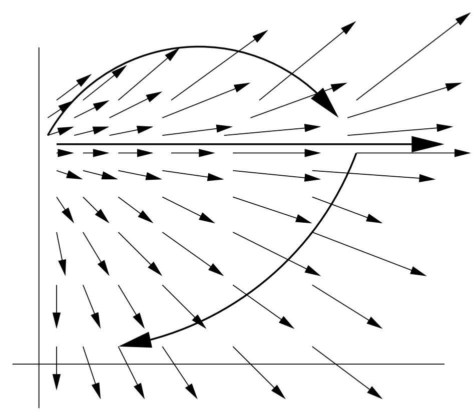
Mathematics 2443-005 Name (please print) ____________________

Examination III Form A

April 10, 2003

I. Calculate the curl and the divergence of the vector field $x^2\vec{\imath} + y^2\vec{\jmath} - xyz\,\vec{k}$.
(6)

II. The figure to the right shows a vector field $\vec{F} = P\vec{\imath} + Q\vec{\jmath}$ and three oriented arcs.
(6)



1. Near each arc, write a small "+" if the line integral of $\vec{F}$ along that arc appears to be positive, a "−" if it appears to be negative, and a "0" if it appears to be 0.
2. Does it appear that $\dfrac{\partial P}{\partial x}$ is positive, negative, or 0?
3. Does it appear that $\dfrac{\partial Q}{\partial y}$ is positive, negative, or 0?
4. Does it appear that $\operatorname{div}(\vec{F})$ is positive, negative, or 0?

III. Use Green's Theorem to calculate $\int_C 3xy\,dx + 5x^2y^2\,dy$, where C is the triangle with vertices $(0,0)$, $(1,0)$, and $(1,1)$.
(5)

IV. (9) Let C be the portion of the circle of radius 2 with center at the origin that lies in the first quadrant $x \geq 0$, $y \geq 0$. By direct calculation using a parameterization of C, evaluate the following line integrals.

1. $\int_C x^2 y \, ds$

2. $\int_C xy \, dy$

3. $\int_C (x\vec{\imath} + y\vec{\jmath}) \cdot d\vec{r}$

V. (4) Use integration to find a function $f(x, y, z)$ for which $\nabla f = (y + z)\vec{\imath} + (x + z)\vec{\jmath} + (x + y)\vec{k}$.

VI. (3) Use the Fundamental Theorem of Calculus to carry out a partial calculation of $\iint_R \frac{\partial P}{\partial x} \, dA$, where R is the rectangle $1 \leq x \leq 3$, $2 \leq y \leq 4$, and $P(x, y)$ is a function of x and y.

VII. (5) Let $f(x, y, z) = \sin(x^2 + y^2 + z)$. Let C_1 be the line segment from $(0, 0, 0)$ to $(1, 1, 0)$, and let C_2 be the curve on the surface $z = e^{xy}$ that lies directly above C_1. Calculate $\int_{C_1} \nabla f \cdot d\vec{r}$ and $\int_{C_2} \nabla f \cdot d\vec{r}$.

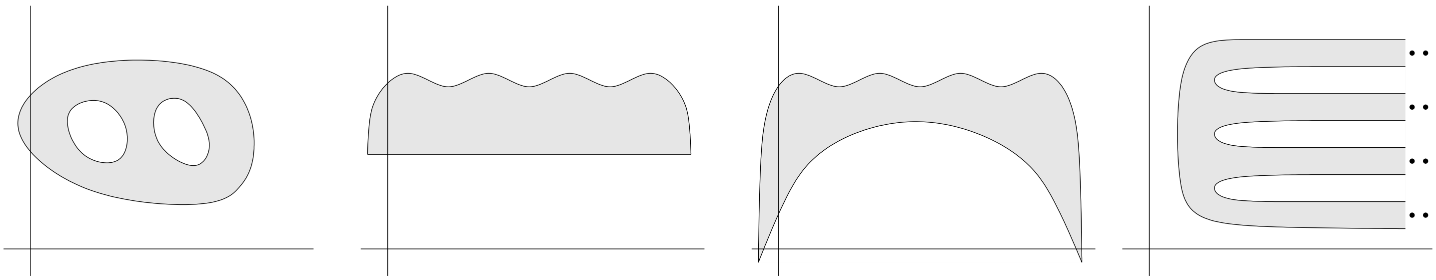
VIII. (7) Let S be the surface given by $x = u\cos(v)$, $y = u\sin(v)$, and $z = u$, where the domain of the parameterization is the rectangle $0 \leq u \leq 1$ and $0 \leq v \leq 2\pi$.

1. Calculate $\vec{r}_u$, $\vec{r}_v$, $\vec{r}_u \times \vec{r}_v$, and $\| \vec{r}_u \times \vec{r}_v \|$.

2. Sketch the domain R in the uv-plane. Tell the points in R where locally the parameterization *neither stretches nor contracts area.*

3. Find an equation in x, y, and z satisfied by all points in the surface (hint: start by calculating $x^2 + y^2$).

IX. (4) The figure below shows four regions in the plane. Below each region, write a very small letter m if the region is simply connected, and a very small letter n if the region is not simply-connected. The three dots on the last region means that the region continues to the right forever.



X. (3) Let C be the unit circle in the xy-plane and let $\vec{T}$ be its unit tangent vector. Suppose that a certain vector field $\vec{F}$ has the property that each point (x, y) in C, $\vec{F} \cdot \vec{T} = \pi$. Find $\int_C \vec{F} \cdot d\vec{r}$.

XI. (2) Give an example of a 2-dimensional vector field $P\vec{\imath} + Q\vec{\jmath}$ which is not conservative but which does satisfy the condition $\dfrac{\partial Q}{\partial x} = \dfrac{\partial P}{\partial y}$. You do not need to verify these properties, just write down the vector field.

XII. (3) Find a vector field $\vec{F}$ in the plane so that if C is any path which does not pass through the origin, and C starts at P and ends at Q, then $\int_C \vec{F} \cdot d\vec{r}$ equals the distance from Q to the origin, minus the distance from P to the origin.