I. Calculate the curl and the divergence of the vector field $x^2\mathbf{i} + y^2\mathbf{j} - xyz\mathbf{k}$.

II. The figure to the right shows a vector field $\vec{F} = P\mathbf{i} + Q\mathbf{j}$ and three oriented arcs.

1. Near each arc, write a small “+” if the line integral of $\vec{F}$ along that arc appears to be positive, a “−” if it appears to be negative, and a “0” if it appears to be 0.

2. Does it appear that $\frac{\partial P}{\partial x}$ is positive, negative, or 0?

3. Does it appear that $\frac{\partial Q}{\partial y}$ is positive, negative, or 0?

4. Does it appear that $\text{div}(\vec{F})$ is positive, negative, or 0?

III. Use Green’s Theorem to calculate $\int_C 3xy\,dx + 5x^2y^2\,dy$, where $C$ is the triangle with vertices (0,0), (1,0), and (1,1).
IV. Let \( C \) be the portion of the circle of radius 2 with center at the origin that lies in the first quadrant \( x \geq 0, y \geq 0 \). By direct calculation using a parameterization of \( C \), evaluate the following line integrals.

1. \( \int_C x^2y \, ds \)

2. \( \int_C xy \, dy \)

3. \( \int_C (x\mathbf{i} + y\mathbf{j}) \cdot d\mathbf{r} \)

V. Use integration to find a function \( f(x, y, z) \) for which \( \nabla f = (y + z)\mathbf{i} + (x + z)\mathbf{j} + (x + y)\mathbf{k} \).

VI. Use the Fundamental Theorem of Calculus to carry out a partial calculation of \( \iint_R \frac{\partial P}{\partial x} \, dA \), where \( R \) is the rectangle \( 1 \leq x \leq 3, 2 \leq y \leq 4 \), and \( P(x, y) \) is a function of \( x \) and \( y \).
VII. Let \( f(x, y, z) = \sin(x^2 + y^2 + z) \). Let \( C_1 \) be the line segment from \((0, 0, 0)\) to \((1, 1, 0)\), and let \( C_2 \) be the curve on the surface \( z = e^{xy} \) that lies directly above \( C_1 \). Calculate \( \int_{C_1} \nabla f \cdot d\vec{r} \) and \( \int_{C_2} \nabla f \cdot d\vec{r} \).

VIII. Let \( S \) be the surface given by \( x = u\cos(v) \), \( y = u\sin(v) \), and \( z = u \), where the domain of the parameterization is the rectangle \( 0 \leq u \leq 1 \) and \( 0 \leq v \leq 2\pi \).

1. Calculate \( \vec{r}_u \), \( \vec{r}_v \), \( \vec{r}_u \times \vec{r}_v \), and \( \| \vec{r}_u \times \vec{r}_v \| \).

2. Sketch the domain \( R \) in the \( uv \)-plane. Tell the points in \( R \) where locally the parameterization *neither stretches nor contracts area*.

3. Find an equation in \( x \), \( y \), and \( z \) satisfied by all points in the surface (hint: start by calculating \( x^2 + y^2 \)).
IX. The figure below shows four regions in the plane. Below each region, write a very small letter \( m \) if the region is simply connected, and a very small letter \( n \) if the region is not simply-connected. The three dots on the last region means that the region continues to the right forever.

X. Let \( C \) be the unit circle in the \( xy \)-plane and let \( \vec{T} \) be its unit tangent vector. Suppose that a certain vector field \( \vec{F} \) has the property that each point \((x, y)\) in \( C \), \( \vec{F} \cdot \vec{T} = \pi \). Find \( \int_C \vec{F} \cdot d\vec{r} \).

XI. Give an example of a 2-dimensional vector field \( P\vec{i} + Q\vec{j} \) which is not conservative but which does satisfy the condition \( \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} \). You do not need to verify these properties, just write down the vector field.

XII. Find a vector field \( \vec{F} \) in the plane so that if \( C \) is any path which does not pass through the origin, and \( C \) starts at \( P \) and ends at \( Q \), then \( \int_C \vec{F} \cdot d\vec{r} \) equals the distance from \( Q \) to the origin, minus the distance from \( P \) to the origin.