I. The figure to the right shows the graph of the polar equation $r = \cos(2\theta)$. Use a double integral in polar coordinates to calculate the area contained inside each one of its loops. You might need to use the identity $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$.

II. Sketch the region in the first octant bounded by the three coordinate planes and the plane $x + y + z = 1$. Write a triple integral whose value is the volume of this region. Supply limits of integration, but do not carry out the calculation to evaluate the integral.

III. Calculate $\int_0^1 \int_0^z \int_0^y z e^{-y^2} \, dx \, dy \, dz$. 

(5) The figure to the right shows the graph of the polar equation $r = \cos(2\theta)$. Use a double integral in polar coordinates to calculate the area contained inside each one of its loops. You might need to use the identity $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$.

(4) Sketch the region in the first octant bounded by the three coordinate planes and the plane $x + y + z = 1$. Write a triple integral whose value is the volume of this region. Supply limits of integration, but do not carry out the calculation to evaluate the integral.

(4) Calculate $\int_0^1 \int_0^z \int_0^y z e^{-y^2} \, dx \, dy \, dz$. 

IV. For the rectangle \( R = [0, 1] \times [0, 2] \), calculate \( \iint_R \frac{xy}{\sqrt{2 + x^2 + y^2}} \, dA. \)

V. The figure to the right shows the portion of the graph of a certain function \( f(x, y) \), and a certain point \( P \) in the domain of \( f \). Also shown are the vector \( \vec{j} \), located at \( P \), and a vector \( \vec{v}_y \) tangent to the surface at the point directly above \( P \). Suppose that \( f_x \) has the value \(-0.65\) at \( P \) and \( f_y \) has the value \(-0.67\). Find \( a, b, \) and \( c \) so that \( \vec{v}_y = a \vec{i} + b \vec{j} + c \vec{k} \).

VI. For the following integral, sketch the region of integration and change the order of integration. The answer should have two terms. \( \int_0^1 \int_{2y}^{4y} f(x, y) \, dx \, dy \)
VII. State the Fundamental Theorem of Calculus (without hypotheses, just the formula). Calculate
\[ \frac{\partial}{\partial x} \int_0^{x^2 + y^2} \sin^{100}(t^2) \, dt. \]

VIII. Find the mass of the upper hemisphere \( E \) given by \( x^2 + y^2 + z^2 = a^2, \ z \geq 0 \) if the density function is \( z \). In spherical coordinates, \( x = \rho \cos(\theta) \sin(\phi), \ y = \rho \sin(\theta) \sin(\phi), \ z = \rho \cos(\phi), \) and \( dV = \rho^2 \sin(\phi) \, d\rho \, d\phi \, d\theta \).

IX. Evaluate \( \iiint_E \sqrt{x^2 + y^2} \, dV \), where \( E \) is the region that lies inside the cylinder \( x^2 + y^2 = 4 \) and between the planes \( z = -1 \) and \( z = 2 \). Use cylindrical coordinates, so that \( \sqrt{x^2 + y^2} = r \).
X. Consider a lamina that occupies the region of the unit disk in the \( xy \)-plane. Suppose that the density at each point is proportional to the cube of the distance from the point to the origin. Write an expression for the density function \( \rho \) in polar coordinates, and use it to find the mass of the lamina.

XI. Use a Riemann sum for this partition of the rectangle \( R = [0, 2] \times [0, 2] \) to estimate \( \iint_{R} \sqrt{x^2 + y^2} \, dA \), choosing as the sample points the points closest to the origin. Leave the Riemann sum as an unsimplified sum of terms, possibly involving square roots.

XII. Consider the paraboloid \( z = x^2 + y^2 \) and the saddle surface \( z = x^2 - y^2 \). Tell how one can know that if \( D \) is any domain in the \( xy \)-plane, then the areas of the portions of these two surfaces having their \((x, y)\) coordinates in \( D \) are equal.