The figure to the right shows the graph of the polar equation \( r = \cos(2\theta) \). Use a double integral in polar coordinates to calculate the area contained inside each one of its loops. You might need to use the identity \( \cos^2(x) = \frac{1}{2} + \frac{1}{2} \cos(2x) \).

Sketch the region in the first octant bounded by the three coordinate planes and the plane \( x + y + z = 1 \). Write a triple integral whose value is the volume of this region. Supply limits of integration, but do not carry out the calculation to evaluate the integral.

Calculate \( \int_0^1 \int_0^z \int_0^y z e^{-y^2} \, dx \, dy \, dz \).
IV. For the rectangle $R = [0, 1] \times [0, 2]$, calculate $\iint_{R} \frac{xy}{\sqrt{2 + x^2 + y^2}} \, dA$.

(4)

V. The figure to the right shows the portion of the graph of a certain function $f(x, y)$, and a certain point $P$ in the domain of $f$. Also shown are the vector $\vec{j}$, located at $P$, and a vector $\vec{v}_y$ tangent to the surface at the point directly above $P$. Suppose that $f_x$ has the value $-0.65$ at $P$ and $f_y$ has the value $-0.67$. Find $a$, $b$, and $c$ so that $\vec{v}_y = a\vec{i} + b\vec{j} + c\vec{k}$.

(3)

VI. For the following integral, sketch the region of integration and change the order of integration. The answer should have two terms. $\int_{0}^{1} \int_{2y}^{4y} f(x, y) \, dx \, dy$.

(5)
VII. State the Fundamental Theorem of Calculus (without hypotheses, just the formula). Calculate
\[ \frac{\partial}{\partial x} \int_0^{x^2y^2} \sin^{100}(t^2) \, dt. \]

VIII. Find the mass of the upper hemisphere \( E \) given by \( x^2 + y^2 + z^2 = a^2 \), \( z \geq 0 \) if the density function is \( z \). In spherical coordinates, \( x = \rho \cos(\theta) \sin(\phi) \), \( y = \rho \sin(\theta) \sin(\phi) \), \( z = \rho \cos(\phi) \), and \( dV = \rho^2 \sin(\phi) \, d\rho \, d\phi \, d\theta \).

IX. Evaluate \( \iiint_E \sqrt{x^2 + y^2} \, dV \), where \( E \) is the region that lies inside the cylinder \( x^2 + y^2 = 4 \) and between the planes \( z = -1 \) and \( z = 2 \). Use cylindrical coordinates, so that \( \sqrt{x^2 + y^2} = r \).
X. Consider a lamina that occupies the region of the unit disk in the $xy$-plane. Suppose that the density at each point is proportional to the cube of the distance from the point to the origin. Write an expression for the density function $\rho$ in polar coordinates, and use it to find the mass of the lamina.

XI. Use a Riemann sum for this partition of the rectangle $R = [0, 2] \times [0, 2]$ to estimate $\iint_R \sqrt{x^2 + y^2} \, dA$, choosing as the sample points the points closest to the origin. Leave the Riemann sum as an unsimplified sum of terms, possibly involving square roots.

XII. Consider the paraboloid $z = x^2 + y^2$ and the saddle surface $z = x^2 - y^2$. Tell how one can know that if $D$ is any domain in the $xy$-plane, then the areas of the portions of these two surfaces having their $(x, y)$ coordinates in $D$ are equal.