Mathematics 2443-003 Name (please print) ______________________

Examination II Form A

March 6, 2003

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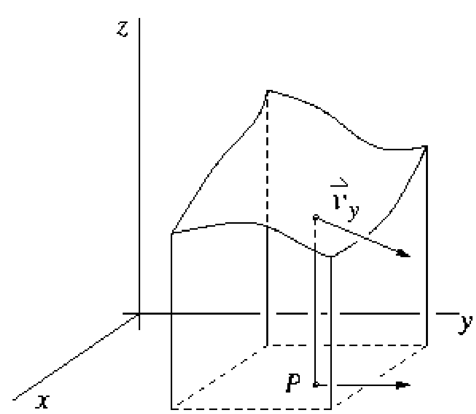
**I.** (5) For the following integral, sketch the region of integration and change the order of integration. The answer should have two terms. $\displaystyle\int_0^1 \int_{2x}^{4x} f(x,y)\,dy\,dx$

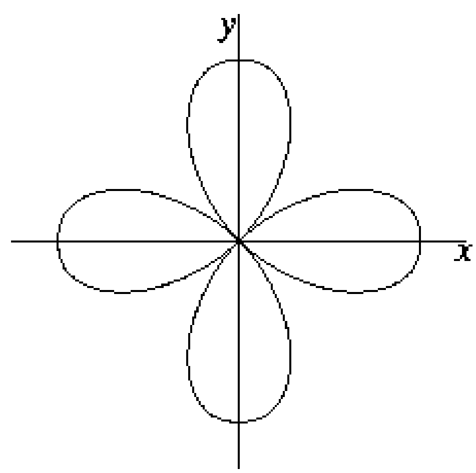
**II.** (4) For the rectangle $R = [0,1] \times [0,2]$, calculate $\iint_R \dfrac{xy}{\sqrt{1+x^2+y^2}}\,dA$.

**III.** (3) The figure to the right shows the portion of the graph of a certain function $f(x,y)$, and a certain point $P$ in the domain of $f$. Also shown are the vector $\vec{\jmath}$, located at $P$, and a vector $\vec{v}_y$ tangent to the surface at the point directly above $P$. Suppose that $f_x$ has the value $-0.67$ at $P$ and $f_y$ has the value $-0.65$. Find $a$, $b$, and $c$ so that $\vec{v}_y = a\vec{\imath} + b\vec{\jmath} + c\vec{k}$.



**IV.** (4) Calculate $\int_0^1 \int_0^z \int_0^y z e^{-y^2}\, dx\, dy\, dz$.

**V.** (5) The figure to the right shows the graph of the polar equation $r = \cos(2\theta)$. Use a double integral in polar coordinates to calculate the area contained inside each one of its loops. You might need to use the identity $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$.



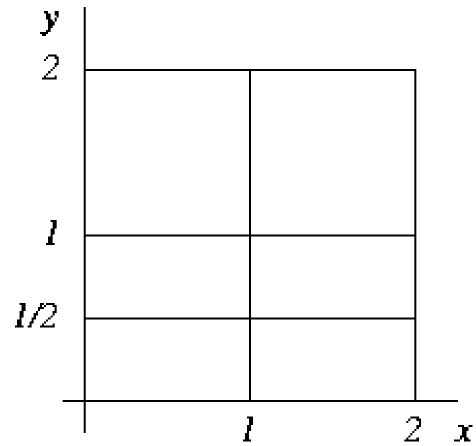

**VI.** (4) Sketch the region in the first octant bounded by the three coordinate planes and the plane $x + y + z = 1$. Write a triple integral whose value is the volume of this region. Supply limits of integration, but *do not* carry out the calculation to evaluate the integral.

**VII.** (5) Find the mass of the upper hemisphere $E$ given by $x^2 + y^2 + z^2 = a^2$, $z \geq 0$ if the density function is $z$. In spherical coordinates, $x = \rho\cos(\theta)\sin(\phi)$, $y = \rho\sin(\theta)\sin(\phi)$, $z = \rho\cos(\phi)$, and $dV = \rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$.

**VIII.** (4) State the Fundamental Theorem of Calculus (without hypotheses, just the formula). Calculate $\dfrac{\partial}{\partial y}\displaystyle\int_0^{x^2y^2} \sin^{100}(t^2)\,dt$.

**IX.** (4) Evaluate $\iiint_E \sqrt{x^2+y^2}\,dV$, where $E$ is the region that lies inside the cylinder $x^2 + y^2 = 4$ and between the planes $z = -1$ and $z = 2$. Use cylindrical coordinates, so that $\sqrt{x^2+y^2} = r$.

**X.** (4) Use a Riemann sum for this partition of the rectangle $R = [0,2] \times [0,2]$ to estimate $\iint_R \sqrt{x^2+y^2}\, dA$, choosing as the sample points the points closest to the origin. Leave the Riemann sum as an unsimplified sum of terms, possibly involving square roots.




**XI.** (5) Consider a lamina that occupies the region of the unit disk in the $xy$-plane. Suppose that the density at each point is proportional to the square of the distance from the point to the origin. Write an expression for the density function $\rho$ in polar coordinates, and use it to find the mass of the lamina.

**XII.** (4) Consider the paraboloid $z = x^2 + y^2$ and the saddle surface $z = x^2 - y^2$. Tell how one can know that if $D$ is any domain in the $xy$-plane, then the areas of the portions of these two surfaces having their $(x, y)$ coordinates in $D$ are equal.