Mathematics 3113-005 Name (please print) ______________________

Examination III

April 18, 2002

I. Calculate the Laplace transforms of the following functions $f(t)$, following any special instructions given.
(15) Make use of the table of formulas whenever possible.

1. $f(t) = t\sinh(t)$

2. $f(t) = \sin(t)\cos(t)$

3. $f(t) = \dfrac{\cos(3t)}{e^{3t}}$.

4. $f(t)$, where $f(t)$ is the second derivative of a function $g(t)$ whose Laplace transform is $\dfrac{\cos^3(s)}{s^3}$, $g(0) = 3$, and $g'(0) = 17$.

5. $f(t) = t$ for $0 \leq t \leq 3$ and $f(t) = 3$ for $t \geq 3$. Use the definition of the Laplace transform.

II. (9) Calculate the *inverse* Laplace transforms of the following functions of s. Follow any special instructions given, but *do not* use partial fractions for any of them— use use other transform methods and formulas.

1. $\dfrac{1}{s(s^2+1)}$. Use the convolution formula and calculate the convolution.

2. $\dfrac{1}{s(s^2+1)}$. Use an integral (different from the convolution) to calculate the inverse transform.

3. $\dfrac{1}{(s^2+7)^2}$.

III. (5) Write the following rational function as a sum of partial fractions, and calculate the unknown coefficients:

$$\frac{s^3+s^2}{(s^2+4)^2}$$

IV. Consider the following system of differential equations:
(5)

$$x''' = y' + 2$$
$$y''' = x' + y + \cos(t)$$

1. Rewrite the system using differential operator notation, so that it is a system of two linear equations in the unknowns x and y, and the coefficients are expressions in the differential operator D.

2. Use Cramer's rule to write a linear differential equation whose solution is x, but do *not* try to solve for x.

V. Use the Laplace transform to solve the following initial value problem:
(6)

$$x'' + 8x' + 15x = 0, \quad x(0) = 0, \; x'(0) = 2$$

VI. (4) Consider the following system of differential equations:

$$x'' = 4x' + y' + 2$$
$$y'' = x + y' + \cos(t)$$

Rewrite the system as a system of four first-order differential equations, in four unknown functions, but do *not* proceed further with solving the system.

VII. (4) Use the Laplace transform and the convolution to solve the first-order linear initial value problem $x' + ax = q(t)$, $x(0) = b_0$, where a is a constant. The answer will be an expression for $x(t)$ that contains an integral involving the unknown function $q(t)$.

VIII. (4) Assuming the (difficult) fact that $\dfrac{d}{ds}\displaystyle\int_0^\infty G(s,t)\,dt = \int_0^\infty \frac{\partial}{\partial s} G(s,t)\,dt$, verify the formula $\dfrac{d}{ds}\mathcal{L}(f(t)) = -\mathcal{L}(t\,f(t))$.