I. (a) Write a linear differential equation whose general solution is $c_1 e^{2x} + c_2 x e^{2x}$.
(b) Write a linear differential equation whose general solution is $c_1 + c_2 e^{2x} \sin(x) + c_3 e^{2x} \cos(x)$.

II. According to the general formula for the method of undetermined coefficients, a trial solution for the linear differential equation $y'' + y = \cos(x)$ is $y = Ax \cos(x) + Bx \sin(x)$. Using this as the trial solution, carry out the method to find the coefficients $A$ and $B$. 
III. Using the formula \( x^s((A_0 + A_1x + \cdots + A_mx^m)e^{rx}\cos(kx) + (B_0 + B_1x + \cdots + B_mx^m)e^{rx}\sin(kx)) \), write trial solutions for the following equations, but do not substitute them into the equations or proceed further with finding the solution.

1. \( y'' - 9y = x^3e^{3x} \)

2. \( y^{(4)} - 16y = \cos(x) \)

IV. For an \( n^{th} \)-order linear differential equation \( y^{(n)} + p_1(x)y^{(n-1)} + \cdots + p_{n-1}(x)y' + p_n(x)y = f(x) \):

1. Tell what it means to say that the equation is **homogeneous**.

2. State the Principle of Superposition.

3. Assuming that the coefficient functions \( p_1(x), \ldots, p_n(x) \) are continuous, tell the initial conditions, at a point \( x = a \), that guarantee that the equation has a unique solution.

V. Give the definition of the statement that \( n \) functions \( y_1(x), y_2(x), \ldots, y_n(x) \) are **linearly independent**.
VI. The figure to the right shows the graphs of solutions of two second-order linear equations with constant coefficients. One equation is $my'' + c_1 y' + ky = 0$, and the other is $my'' + c_2 y' + ky = 0$, where $m, c_1, c_2$ and $k$ are four positive numbers.

1. Which of the graphs shows the solution of an equation with more damping, the first or the second?

2. If $c_1 < c_2$, which of the graphs shows a solution to $my'' + c_1 y' + ky = 0$, the first or the second?

VII. Use the method of variation of parameters to find a particular solution of the differential equation $y'' - y = e^x$ as follows.

1. Find two linearly independent solutions $y_1(x)$ and $y_2(x)$ of the associated homogeneous equation.

2. Use the general equations $u'_1 y_1 + u'_2 y_2 = 0$, $u'_1 y'_1 + u'_2 y'_2 = f(x)$ of the method of variation of parameters to find $u'_1$ and $u'_2$.

3. Use the result of the previous step to find a particular solution.
VIII. For the boundary value problem \( y'' + 2y' + \lambda y = 0 \), \( y(0) = y(1) = 0 \), show that \( \lambda = 1 \) is not an eigenvalue. 

IX. For the boundary value problem \( y'' + \lambda y = 0 \), \( y(0) = y(1) = 0 \), find all positive eigenvalues, and an associated eigenfunction for each of the positive eigenvalues. Show your work.

X. Rewrite \( 2 \cos(3x) + 7 \sin(7x) \) in phase-angle form. Give the exact function, not a decimal approximation (so your answer will involve an inverse tangent function).