Mathematics 3113-005 Name (please print) ______________________

Examination II

March 14, 2002

I. (a) Write a linear differential equation whose general solution is $c_1 e^{2x} + c_2 x e^{2x}$.
(6)

(b) Write a linear differential equation whose general solution is $c_1 + c_2 e^{2x} \sin(x) + c_3 e^{2x} \cos(x)$.

II. According to the general forumula for the method of undetermined coefficients, a trial solution for the
(6) linear differential equation $y'' + y = \cos(x)$ is $y = Ax\cos(x) + Bx\sin(x)$. Using this as the trial solution, carry out the method to find the coefficients A and B.

III. (6) Using the formula $x^s\big((A_0 + A_1x + \cdots + A_mx^m)e^{rx}\cos(kx) + (B_0 + B_1x + \cdots + B_mx^m)e^{rx}\sin(kx)\big)$, write trial solutions for the following equations, but *do not* substitute them into the equations or proceed further with finding the solution.

1. $y'' - 9y = x^3e^{3x}$

2. $y^{(4)} - 16y = \cos(x)$

IV. (6) For an n^{th}-order linear differential equation $y^{(n)} + p_1(x)y^{(n-1)} + \cdots + p_{n-1}(x)y' + p_n(x)y = f(x)$:

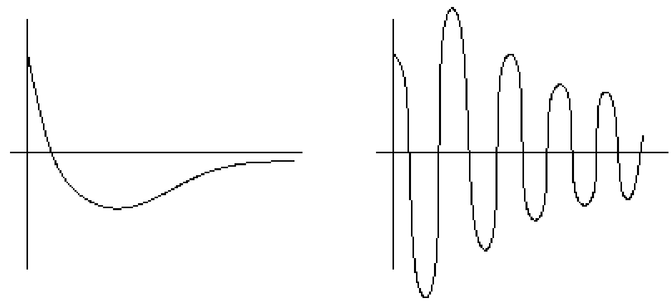
1. Tell what it means to say that the equation is *homogeneous.*

2. State the Principle of Superposition.

3. Assuming that the coefficient functions $p_1(x), \ldots, p_n(x)$ are continuous, tell the initial conditions, at a point $x = a$, that guarantee that the equation has a unique solution.

V. (3) Give the definition of the statement that n functions $y_1(x)$, $y_2(x), \ldots, y_n(x)$ are *linearly independent.*

VI. (4) The figure to the right shows the graphs of solutions of two second-order linear equations with constant coefficients. One equation is $my'' + c_1y' + ky = 0$, and the other is $my'' + c_2y' + ky = 0$, where m, c_1, c_2 and k are four positive numbers.



1. Which of the graphs shows the solution of an equation with more damping, the first or the second?

2. If $c_1 < c_2$, which of the graphs shows a solution to $my'' + c_1y' + ky = 0$, the first or the second?

VII. (8) Use the method of variation of parameters to find a particular solution of the differential equation $y'' - y = e^x$ as follows.

1. Find two linearly independent solutions $y_1(x)$ and $y_2(x)$ of the associated homogeneous equation.

2. Use the general equations $u_1'y_1 + u_2'y_2 = 0$, $u_1'y_1' + u_2'y_2' = f(x)$ of the method of variation of parameters to find u_1' and u_2'.

3. Use the result of the previous step to find a particular solution.

VIII. For the boundary value problem $y'' + 2y' + \lambda y = 0$, $y(0) = y(1) = 0$, show that $\lambda = 1$ is not an eigenvalue.
(4)

IX. For the boundary value problem $y'' + \lambda y = 0$, $y(0) = y(1) = 0$, find all *positive* eigenvalues, and an
(6) associated eigenfunction for each of the positive eigenvalues. Show your work.

X. Rewrite $2\cos(3x) + 7\sin(7x)$ in phase-angle form. Give the exact function, not a decimal approximation
(4) (so your answer will involve an inverse tangent function).