Mathematics 2443-006H — Name (please print) ____________________

Examination II

March 28, 2001

I. (6) Calculate the iterated integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \dfrac{1}{\sqrt{1+x^2+y^2}}\, dy\, dx$.

II. (6) Calculate the iterated integral $\int_0^1 \int_0^1 \dfrac{xy}{\sqrt{1+x^2+y^2}}\, dy\, dx$.

III. (6) Let r represent a number greater than 1. Find the y-coordinate of the centroid of the triangle with vertices $(0,0)$, $(1,1)$, $(1,r)$ (note that the area of this triangle is $\dfrac{r-1}{2}$).

IV. (5) Use facts about the cross-product to verify that the area of the parallelogram in the xy-plane determined by the vectors $a\vec{\imath}+b\vec{\jmath}$ and $c\vec{\imath}+d\vec{\jmath}$ is $|ad-bc|$.

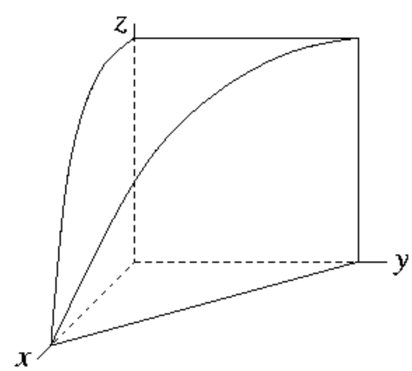
V. (10) Let R be the region in the xy-plane bounded by the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$.

1. Verify that the transformation $x=au$, $y=bv$ sends the unit disc S in the uv-plane to the region R.
2. Calculate the Jacobian matrix for this change of variable, and its determinant $\dfrac{\partial(x,y)}{\partial(u,v)}$.
3. Use this change of variable to calculate $\iint_R x^2\, dA$.

VI. (6) Evaluate by reversing the order of integration: $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\, dy\, dx$.

VII. (8) The figure to the right shows the region of integration for the integral $\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z)\, dy\, dz\, dx$.

1. Supply new limits if the order of integration is $dz\, dy\, dx$.
2. Supply new limits if the order of integration is $dx\, dy\, dz$.



VIII. (6) Evaluate $\iiint_E \sqrt{x^2+y^2+z^2}\, dV$, where V is the region in the first octant that lies between the spheres $\rho=2$ and $\rho=4$ and above the cone $\phi=\pi/6$.

IX. (5) Let D be a region in the xy-plane, of area $A(D)$. Show that the area of the portion of the plane $z=ax+by$ lying in the vertical cylinder determined by D is $\sqrt{a^2+b^2+1}\, A(D)$.

X. (6) Let $f(x,y)=x^2y^2$ and let R be the rectangle $0\le x\le 2$, $0\le y\le 2$. Subdivide R into four equal squares. For this partition of R, find the largest and smallest Riemann sums for $f(x,y)$.