Mathematics 3113-002 Name (please print) ______________________

Examination III

April 27, 2001

I. (5) Use the convolution to write the general solution of the initial value problem $x'' + x = f(t)$, $x(0) = 0$, $x'(0) = 0$ as an integral of an expression that involves the function f. (Of course, not knowning f explicitly, one cannot proceed further with solving the equation.)

II. (5) Write the equation $x^{(3)} - x' = e^{2t}$ as a system of first-order equations, but do *not* try to proceed further with finding its solutions.

III. (5) Write the rational function $\dfrac{s^3}{(s^2+3)^2(s^2-3)^2}$ as an appropriate sum of partial fractions whose numerators contain unknown constants, but do *not* try to solve for those unknown constants.

IV. Calculate the *inverse* Laplace transforms of the following functions of s, following any special instructions given. Note: partial fractions calculations are *not* used for any of them. All are to be done using other transform methods and formulas.
(16)

1. $\dfrac{1}{s^4}$

2. $\dfrac{1}{s^4(s^2+4)}$ (use the expression for the inverse transform of $\dfrac{1}{s^4}$ found in the previous part, together with the convolution, to write the inverse transform as an integral, but do *not* calculate the integral).

3. $\dfrac{s}{(s^2+1)^2}$ (use the fact that $\dfrac{d}{ds}\left(\dfrac{1}{s^2+1}\right) = -\dfrac{2s}{(s^2+1)^2}$).

4. $\dfrac{10s-3}{25-s^2}$

V. Consider the following system of differential equations
(6)

$$\begin{aligned} x' &= 4x + y + 2\sin(t) \\ y' &= x' + y \end{aligned}$$

1. Rewrite the system using differential operator notation.

2. Use Kramer's rule to write a linear differential equation whose solution is x, and a linear differential equation whose solution is y, but do *not* try to solve for x and y.

VI. For the following initial value problem, transform the equation and solve for the transform $X(s)$ of $x(t)$, but
(5) do *not* set up partial fractions or otherwise attempt to calculate the inverse transform: $x^{(3)} + 6x' - 5x = 2t$, $x(0) = 0$, $x'(0) = -1$, $x''(0) = 0$.

VII. Calculate the Laplace transforms of the following functions of t.
(10)

1. $\dfrac{\sin t}{t}$

2. $f(t) = \begin{cases} 1 & \text{if } 2na \leq t \leq (2n+1)a,\ n = 0, 1, 2, \ldots \\ 0 & \text{if } (2n+1)a < t < (2n+2)a,\ n = 0, 1, 2, \ldots \end{cases}$