Mathematics 3113-002 Name (please print) ______________________

Examination I

February 12, 2001

I. Find general or implicit solutions of the following differential equations.

(12)

1. $xy' = 2y + x^3\cos(x)$ (assume that $x > 0$)

2. $x = 2\sqrt{x^2 - 16}\; y\,y'$ (assume that $x > 4$)

II. (5) A general solution to the differential equation $y' = x - y$ is $y = Ce^{-x} + x - 1$. Solve the initial value problem $y' = x - y$, $y(0) = 10$.

III. (3) What information does the Existence and Uniqueness Theorem give about the initial value problem $xy' = \sqrt[3]{y} + x^3 \cos(x)$, $y(1) = 0$?

IV. (3) What information does the Existence and Uniqueness Theorem give about the initial value problem $xy' = \sqrt[3]{y} + x^3 \cos(x)$, $y(1) = 1$?

V. (3) What information does the Existence and Uniqueness Theorem give about the initial value problem $xy' = \sqrt[3]{y} + x^3 \cos(x)$, $y(0) = 0$?

VI. (6) The differential equation $(x + y)y' = x - y$ is known to be exact. Use the method for exact equations to obtain an implicit solution for it.

VII. (6) The half-life of radioactive cobalt is 5.27 years. A sample of radioactive cobalt weighing 100 kilograms is buried in a nuclear waste storage facility. After 200 years, how much cobalt will remain in the sample? (Give the answer in exact form, involving a fractional power of 2.)

VIII. Find a general solution to the differential equation $3y^2y' + y^3 = e^{-x}$.
(6)

IX. A certain object is dropped in the ocean and sinks. Let s be its depth at time t, and let $v = \dfrac{ds}{dt}$. Assume that the net downward force from gravity and buoyancy is a constant ℓ, and that the water resists the motion of the object with a force proportional to $v^{3/2}$.
(6)

1. Write a differential equation of the form $v' = f(v, t)$ that the velocity must satisfy.

2. Separate the variables for this equation, obtaining an equation of the form $t + C = \int f(v)\, dv$. Use the substitution $u^2 = v$ to change the integrand into a rational function of u, but *do not* proceed further with the calculation. (If you had more time, you could carry out the method of partial fractions to calculate the integral and obtain an implicit solution.)