Mathematics 1823-010
Examination III Form A
April 20, 2001

Name (please print) _____

Student Number _____

Discussion Section (please circle day and time): (1)
We 2:30 We 3:30 Th 9:00 Th 10:30 Th 12:00 Th 1:30

I. (6) A certain function $f(x)$ has $f'(x) = 3x^2 - 6$ and $f''(x) = 6x$. Carry out the following.

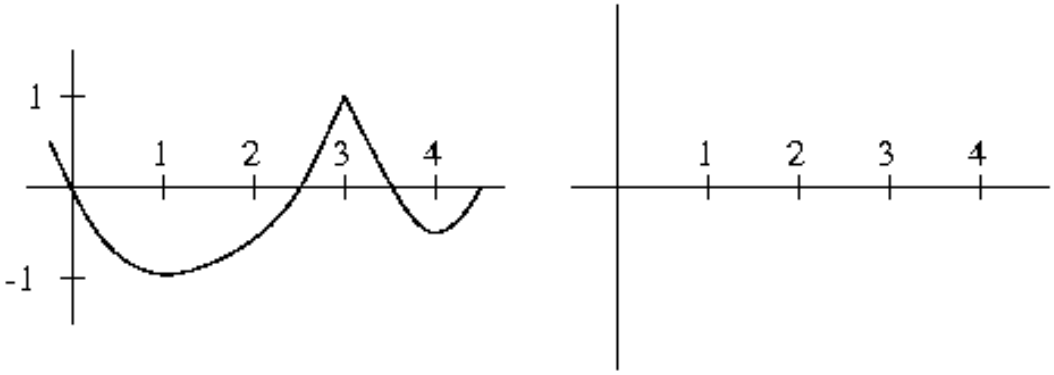
1. Find all critical points of f.

2. By examining $f'(x)$, find all local maxima of f.

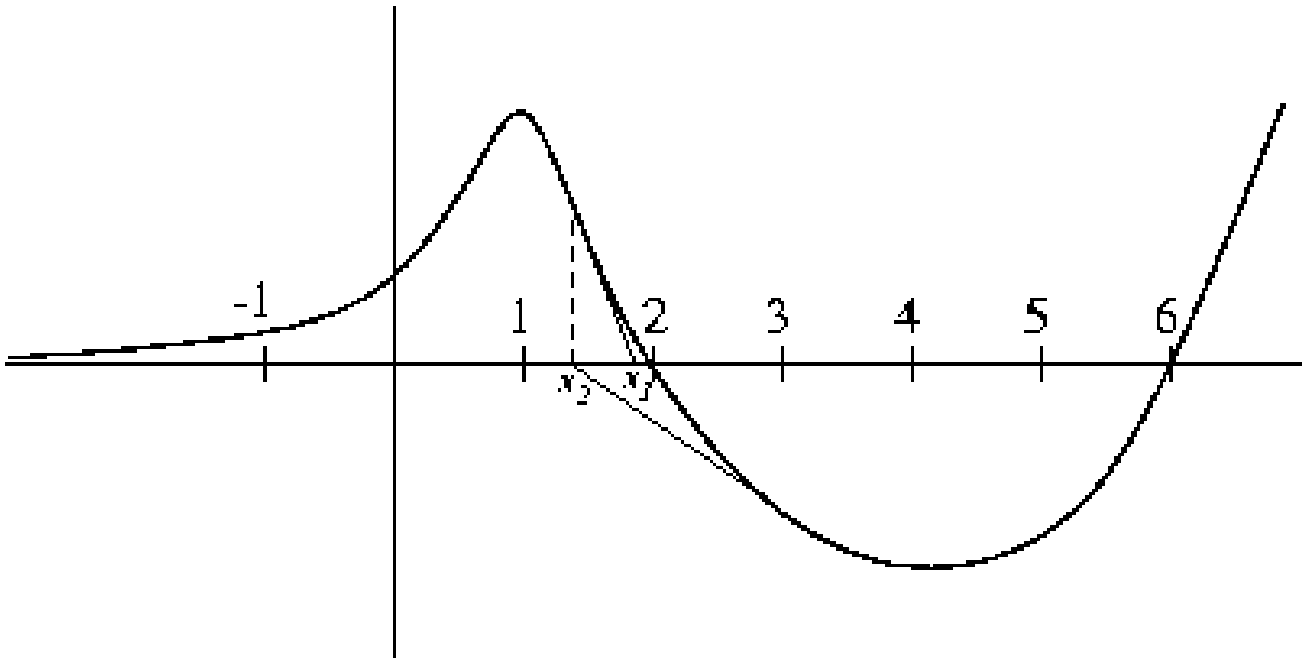
3. By examining $f'(x)$, find all local minima of f.

II. (3) A certain function $g(x)$ has $g''(x) = \dfrac{x}{x+7}$. By examining $g''(x)$, find where g is concave up, and where it is concave down.

III. (5) The first coordinate system below shows the graph of a function $f(x)$. On the second coordinate system, sketch the graph of its derivative $f'(x)$.



IV. (6) The coordinate system shown here is the graph of a certain differentiable function which has roots at $x = 2$ and $x = 6$, and has a local maximum at $x = 1$ and a local minimum at $x = 4$. For each of the choices of the value of x_1 given below, draw (if possible) the appropriate tangent lines and the points x_2 and x_3 that result when Newton's method is carried out using the starting value x_1. For each choice of x_1, tell what the iteration would do if continued. The problem is already worked out for $x_1 = 3$, as an example.



Example: $x_1 = 3$

The iteration would converge to the root $x = 2$.

1. $x_1 = 0$

2. $x_1 = 1$

3. $x_1 = 5$

V. (6) For the function $f(x) = \dfrac{(1-x^2)(1-x)}{x^3}$, carry out the following.

1. Determine all vertical asymptotes of the graph of $f(x)$.

2. Determine all horizontal asymptotes of the graph of $f(x)$.

3. Determine all values of x at which $f(x)$ changes sign.

VI. (7) Let $f(x) = \dfrac{x}{x+2}$ on the domain $1 \leq x \leq 4$ (i. e. the closed interval $[1, 4]$). Find all values of c that satisfy the conclusion of the Mean Value Theorem for this function on this interval.

VII. (10) Calculate the following limits. (To obtain credit, you must show your reasoning, not just guess the limit by computing values on a calculator.)

1. $\lim_{x\to\infty} \sqrt{x^2+x} - x$

2. $\lim_{x\to-\infty} \dfrac{(1-x)(1+2x)}{(2+x)(2-3x)}$

VIII. (4) Let $f(x)$ be a function and let a be an x-value in the domain of $f(x)$. Define what it means to say that f has a *local maximum* at $x = a$, and define what it means to say that f has an *absolute maximum* at $x = a$.

IX. (4) State the Mean Value Theorem.