Mathematics 1823-010
Examination I Form B
February 16, 2001

Name (please print) ______________________

Student Number ______________

Discussion Section (please circle day and time):
We 2:30 We 3:30 Th 9:00 Th 10:30 Th 12:00 Th 1:30

I. (4) Calculate the limit $\lim_{x\to 4} \dfrac{x-4}{\sqrt{x}-2}$.

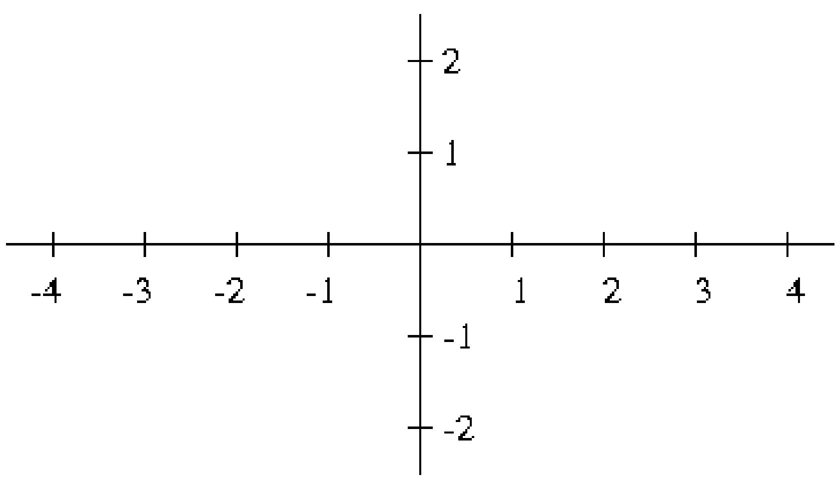
II. (3) Complete this statement of the Intermediate Value Theorem: Suppose that $f(x)$ is a continuous function on the domain $a \leq x \leq b$, and let N be any number ...

III. (7) Use the formula $\lim_{x\to a} \dfrac{f(x)-f(a)}{x-a}$ to calculate the slope of the tangent line to the graph of the function $f(x) = \sqrt{3x+1}$ at the point where $x = a$. (Hint: Use $(A-B)(A+B) = A^2-B^2$ to simplify the numerator.)

IV. (4) Write the precise ϵ-δ definition of the following: $\lim_{f \to g} x(f) = a$.

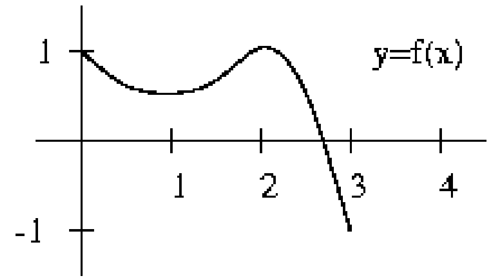
V. (4) Use the definition of limit to prove that $\lim_{x \to 0} x^{1/3} = 0$.

VI. (6) In this coordinate system, graph the functions $x^{1/3}$, $x^{1/4}$, and $x^{1/5}$, indicating which curve corresponds to each function.

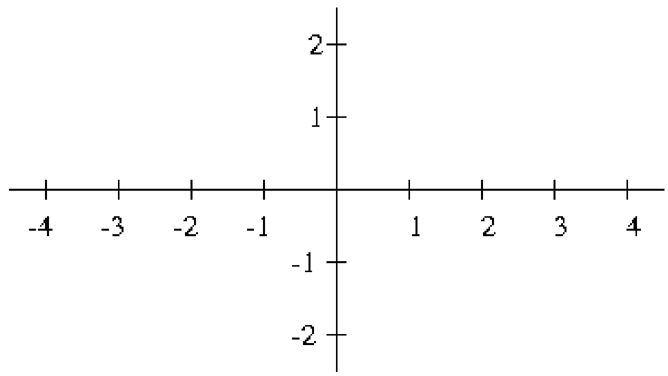


VII. (4) For the functions $f(x) = x^4 + 2x$ and $g(x) = \dfrac{1}{x}$, calculate the compositions $f \circ g$ and $g \circ f$.

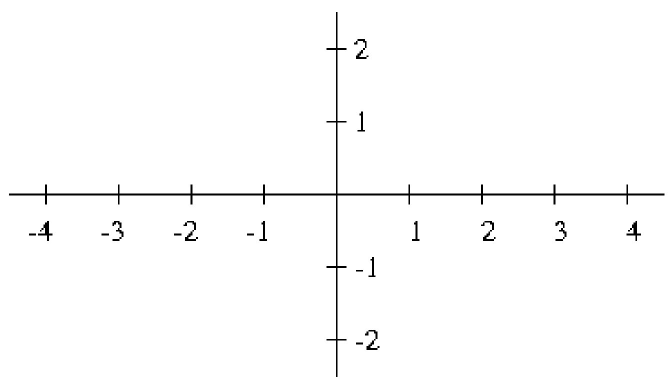
VIII. The problems on this page all refer to the function $f(x)$ whose graph is shown in this coordinate system:
(9)



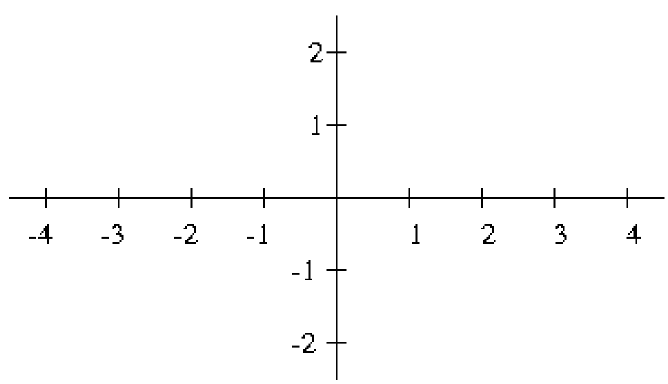
1. In this coordinate system, graph the function $-2f(x)$.



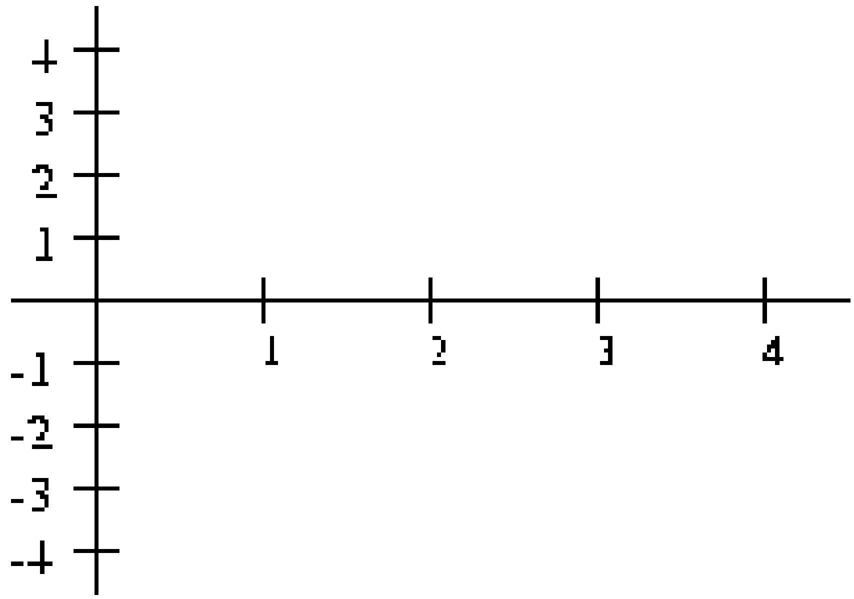
2. In this coordinate system, graph the function $-2f(x+3)$.



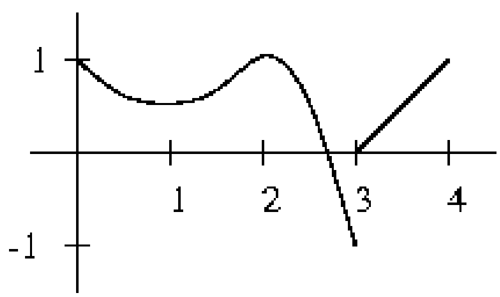
3. In this coordinate system, graph the function $1/f(x)$.

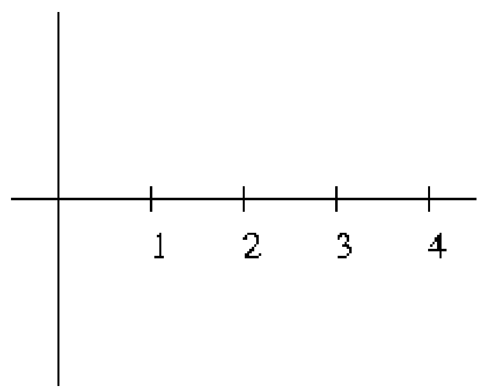


IX. (5) In the coordinate system below, sketch the graph of a function $g(x)$ satisfying the following conditions: $g(0) = 1$, $g'(0) = -1$, g is discontinuous at $x = 1$ but is continuous at all other $x < 4$, $g(2) = 0$, $g'(2) = 3$, $g'(3) = 0$, and $\lim_{x \to 4^-} g(x) = -\infty$.



X. (5) The first coordinate system shows the graph of a function $f(x)$. On the second coordinate system, sketch the graph of its derivative $f'(x)$.





XI. (3) (bonus problem) Sketch the graph of $y = \frac{1}{x} \sin\left(\frac{1}{x}\right)$.