Mathematics 1823-010
Final Examination Form B
May 7, 2001

Name (please print) ______________________

Student Number ______________

I. Give a precise mathematical definition of each of the following:
(7)

1. $x = a$ is a *critical point* (also called a *critical number*) of $f(x)$

2. $\lim\limits_{f \to L} g(f) = a$ (give the ϵ-δ definition)

3. $f(x)$ has a *local minimum* at $x = a$ (use an open interval to express the idea that x is near a)

II. For each of the following, find *all* $f(x)$ that satisfy the given condition or conditions.
(9)

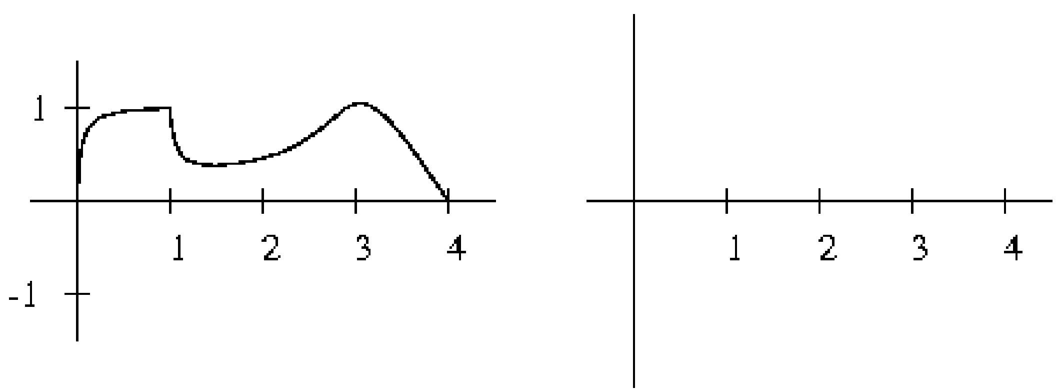
1. $f'(x) = \pi x^2 - 4x^3$

2. $f''(x) = x$

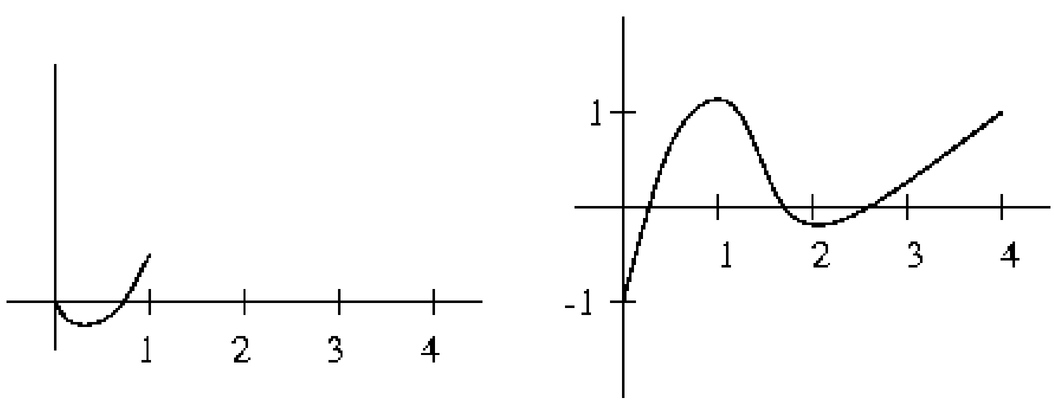
3. $f'(x) = 4 + \sin(x)$ and $f(0) = 2$

III. Use the definition of limit to prove that $\lim_{x \to 2} 5x + 1 = 11$.
(4)

IV. The first coordinate system shows the graph of a function $f(x)$. On the second coordinate system, sketch
(4) the graph of its derivative $f'(x)$.



V. The portion of the graph of a function $g(x)$ for $0 \leq x \leq 1$ is sketched on the first coordinate system below.
(4) The second coordinate system shows the graph of the derivative $g'(x)$. On the first coordinate system, sketch the remaining part of the graph of $g(x)$.



VI. (12) A table giving some of the values for f, g, f', and g' is given here:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	-1	2	2	4
2	1	8	3	0
3	5	2	1	9

For each of the following questions, please **circle the letter** of the correct answer, based on the values of f and g given in the table.

1. If $h(x) = f(g(x))$, find $h'(1)$.

   (a) -3 (b) 0 (c) 2
   (d) 3 (e) 5 (f) 6
   (g) 9 (h) 12 (i) 15

2. If $f''(1) = 4$, $f''(2) = 6$, and $f''(3) = 2$, what is the value of the derivative of $f(f'(x))$ at $x = 2$?

   (a) 2 (b) 3 (c) 4
   (d) 6 (e) 8 (f) 12
   (g) 16 (h) 20 (i) 21

3. If f is differentiable, then (based on the values of f given in the table) the Mean Value Theorem guarantees that $f'(x)$ must assume which one of the following values:

   (a) 7 (b) 1 (c) 6
   (d) -1 (e) 12 (f) 4
   (g) 0 (h) 18 (i) 21

4. Based on the values given in the table, the linear part of the change of g as x changes from 2 to 3 is:

   (a) -9 (b) -6 (c) -1
   (d) 0 (e) 3 (f) 4
   (g) 6 (h) 8 (i) 9

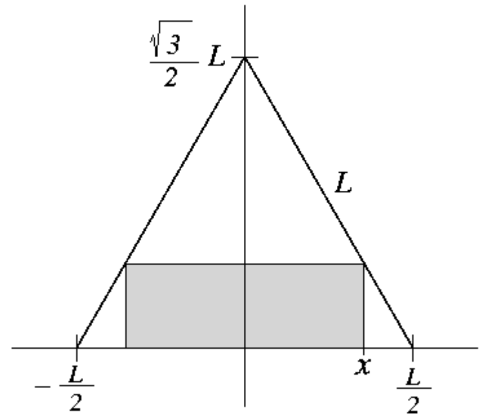
5. If f is continuous, then it must have a root between $x = 0$ and $x = 1$. One can conclude this using which of the following?

   (a) Newton's Method (b) Rolle's Theorem (c) The Mean Value Theorem
   (d) The Fundamental Theorem of Algebra (e) The Intermediate Value Theorem (f) Linear Approximation
   (g) The Second Derivative Test (h) The Extreme Value Theorem (i) The First Derivative Test

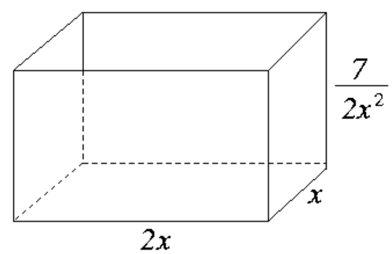
6. If f is continuous, then it must assume a minimum value on the closed interval $[0, 1]$. One can conclude this using which of the following?

   (a) Newton's Method (b) Rolle's Theorem (c) The Mean Value Theorem
   (d) The Fundamental Theorem of Algebra (e) The Intermediate Value Theorem (f) Linear Approximation
   (g) The Second Derivative Test (h) The Extreme Value Theorem (i) The First Derivative Test

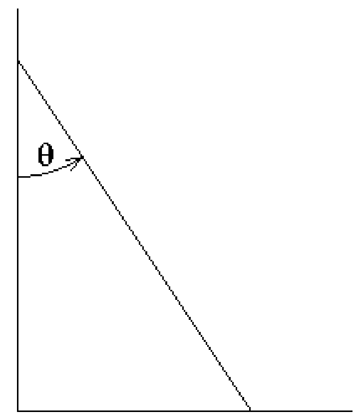
VII. (4) The figure to the right shows an equilateral triangle with sides of length L and a rectangle inscribed in the triangle. Express the area of the rectangle in terms of the x-coordinate of the lower right-hand corner of the rectangle. Do *not* proceed further with the maximization problem, just give the area A as a function of x and no other variables.



VIII. (6) The figure to the right shows a rectangular box with open top. The base has length twice its width. The volume of the box is 7 cubic units. We let x be the width of the base, so that the length of the base is $2x$ and the height is $\frac{7}{2x^2}$. Calculate the surface area S (the total of the 4 sides, plus the base) in terms of x, and find the value of x that minimizes S.



IX. (7) As shown in the figure to the right, a ladder 12 feet long leans against a wall. Its base is sliding directly away from the wall at 3 feet per second. How fast is the angle θ changing when θ is $\pi/4$?



X. (8) For each of the following questions, please **circle the letter** of the correct answer,

1. Suppose that L is a function for which $L'(x) = \dfrac{1}{x}$. The derivative of $(L(x))^2$ is:

(a) $-\dfrac{2L(x)}{x^2}$ (b) $\dfrac{2L(x)}{x^2}$ (c) $\dfrac{2}{x}$

(d) $\dfrac{2}{L(x)}$ (e) $2xL(x)$ (f) $\dfrac{2L(x)}{x}$

(g) $2L(x)$ (h) $\dfrac{1}{x^2}$ (i) $-\dfrac{1}{x^2}$

2. Which of the following can be applied to a function *only* when the function is a polynomial?

(a) Newton's Method (b) Rolle's Theorem (c) The Mean Value Theorem

(d) The Fundamental Theorem of Algebra (e) The Intermediate Value Theorem (f) Linear Approximation

(g) The Second Derivative Test (h) The Extreme Value Theorem (i) The First Derivative Test

3. The derivative of a certain function r is $r'(x) = \dfrac{x}{1+x^2}$. At $x = 0$, r has

(a) an absolute maximum (b) a local maximum which is not an absolute maximum (c) a critical point where r has neither a local maximum nor a local minimum

(d) an absolute minimum (e) a local minimum which is not an absolute minimum (f) a point where the graph of r has vertical slope

(g) an inflection point (h) a vertical asymptote (i) a horizontal asymptote

XI. Calculate the following derivatives.
(12)

1. $\dfrac{(x^3+8)^5}{\cos(x)}$

2. $f''(x)$ if $x = \tan(x)$

3. $\dfrac{dy}{dx}$ if $y^2 = \sin(x+y)$ (use implicit differentiation)