Mathematics 2443-004 | Name (please print) ______________________
Examination III Form B | Student Number ______________
April 17, 2000

I. Let C be the portion of the circle of radius 2 from $(2,0)$ to $(0,2)$, oriented counterclockwise.
(11)

1. Write a parameterization of C.

2. Evaluate $\int_C x\vec{\imath} \cdot d\vec{r}$, by direct calculation from the definition of $\int_C \vec{F} \cdot d\vec{r}$.

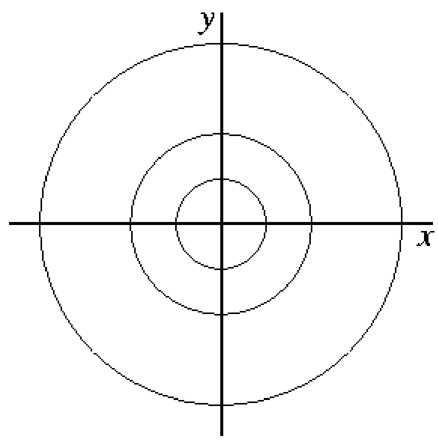
3. Evaluate $\int_C x\vec{\imath} \cdot d\vec{r}$, using the Fundamental Theorem for Line Integrals.

II. Suppose that $\vec{F}$ is a vector field on a 3-dimensional domain. For each of the following, state whether the expression represents a scalar field (i. e. a function), a vector field, or is meaningless.
(6)

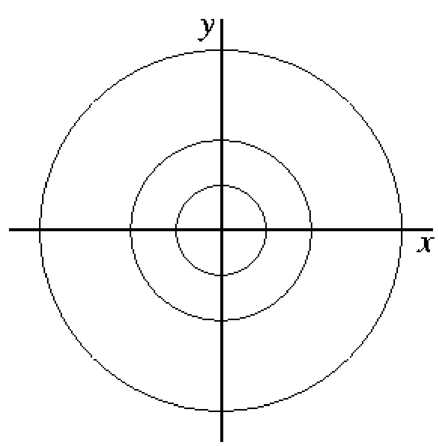
1. $\mathrm{curl}(\mathrm{curl}(\vec{F}))$
2. $\mathrm{div}(\mathrm{div}(\vec{F}))$
3. $\mathrm{div}(\mathrm{curl}(\vec{F}))$
4. $\mathrm{curl}(\mathrm{div}(\vec{F}))$

III. (6) The coordinate systems below show the circles of radius 1/2, 1, and 2 centered at the origin. For each of the following vector fields, sketch enough of the vectors on these circles to indicate what the vector field is like.

1. $x\vec{\imath} + y\vec{\jmath}$



2. $y\vec{\imath} - x\vec{\jmath}$ (Hint: $(x\vec{\imath} + y\vec{\jmath}) \cdot (y\vec{\imath} - x\vec{\jmath}) = 0$.)



IV. (7) Use Green's Theorem to calculate $\int_C xy\,dx + x^2\,dy$, where C consists of the line segment from $(-2,0)$ to $(2,0)$ and the top half of the circle $x^2+y^2=4$. (Hint: $dA = r\,dr\,d\theta$.)

V. (7) For the vector field $\vec{F} = \cos(x)\,\vec{\imath} + \sin(z)\,\vec{\jmath} + \tan(y)\,\vec{k}$, calculate $\operatorname{div}(\vec{F})$ and $\operatorname{curl}(\vec{F})$.

VI. (6) Let R be a region in the plane. Define what it means to say that R is *simply-connected.* Explain how the definition shows that the region given in polar coordinates by $1 < r < 2$ is not simply-connected.

VII. (7) The equations $x = a\sin(\phi)\cos(\theta)$, $y = a\sin(\phi)\sin(\theta)$, $z = a\cos(\phi)$ give a parametric representation of the sphere $x^2 + y^2 + z^2 = a^2$.

1. Calculate r_ϕ.

2. On this picture of the sphere, label the lines where θ is constant, and those where ϕ is constant. Draw some of the vectors $\vec{r}_\phi$ and $\vec{r}_\theta$, and also some of the vectors $\vec{r}_\phi \times \vec{r}_\theta$.