Mathematics 2934-010 Name (please print) ______________________

Quiz 5

November 3, 2011

Instructions: Give concise answers, but clearly indicate your reasoning.

I. Consider the solid in the first octant bounded by the three coordinate planes and the plane $3x+2y+z=6$.
(6)

(a) What are the x and y-intercepts of the plane?

The x-intercept is 2 and the y-intercept is 3.

(b) The base of the solid is a triangle in the xy-plane. In an xy-plane, draw a picture of the base, and give equations for its sides.

It is a triangle in the first quadrant, with two sides that are the axes $x = 0$ and $y = 0$, and the hypotenuse is $\dfrac{x}{2} + \dfrac{y}{3} = 1$, or $3x + 2y = 6$.

(c) Write a double integral to find the volume of the solid. Supply specfic limits of integration, but *do not* carry out any further calculations or try to evaluate it.

$\int_0^2 \int_0^{3-3x/2} 6 - 3x - 2y\, dy\, dx$, or $\int_0^3 \int_0^{2-2y/3} 6 - 3x - 2y\, dx\, dy$.

II. A lamina occupies the unit square R, where $0 \le x \le 1$ and $0 \le y \le 1$. Its density at (x, y) is proportional
(6) to x^3. Write definite integrals to calculate each of the following, but *do not* carry out the evaluation of the integrals.

(a) The mass of the lamina.

$\int_0^1 \int_0^1 kx^3\, dy\, dx$.

(b) The moment of the lamina with respect to the x-axis.

$\int_0^1 \int_0^1 kyx^3\, dy\, dx$.

(c) The x-coordinate of the center of mass of the lamina, where m is its mass.

$\frac{1}{m} \int_0^1 \int_0^1 kx^4\, dy\, dx$.

III. Change the order of integration for the following integral, but *do not* carry out any further calculations or
(4) try to evaluate it: $\displaystyle\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx\, dy$.

The domain for the integral is the region bounded by $x = \sqrt[3]{y}$, that is, $y = x^3$, the line $x = 2$, and the line $y = 0$. Reversing the order of integration gives $\displaystyle\int_0^2 \int_0^{x^3} e^{x^4}\, dy\, dx$.

IV. Using polar coordinates, evaluate the integral $\iint_D 2e^{-x^2-y^2}\, dA$, where D is the region bounded by $y =$
(4) $\sqrt{4 - x^2}$ and the x-axis.

The region D can be described by $0 \le r \le 2$, $0 \le \theta \le \pi$. In polar coordinates, the integral becomes $\int_0^\pi \int_0^2 2e^{-r^2} r\, dr\, d\theta = \int_0^\pi \left(-e^{-r^2} \Big|_0^2 \right) d\theta = \int_0^\pi 1 - e^{-4}\, d\theta = \pi(1 - e^{-4})$.