Math 2934 homework

26. (11/4) Suppose you have a function $g(u, v)$ of two variables. Its domain is a region R in the uv-plane. Consider its graph in uvw-space, $w = g(u, v)$.

    (a) For a small rectangle of sides Δu_i and Δv_j in the uv-plane, whose lower left corner has coordinates (u_i, v_j), use the method we discussed in class to verify that in the tangent plane to $w = g(u, v)$ at the point $(u_i, v_j, g(u_i, v_j))$, the area lying above the rectangle has area $\sqrt{1 + g_u(u_i, v_j)^2 + g_v(u_i, v_j)^2}\,\Delta u_i \Delta v_j$.

    (b) The limit of the Riemann sums using the areas calculated in part (a) is $\iint_R \sqrt{1 + g_u(u, v)^2 + g_v(u, v)^2}\,dA$, an expression for the surface area of the graph. Use this to find the area of the portion of the saddle surface $z = u^2 - v^2$ lying over the unit disk.

    (c) Use the surface area formula to find the area of the part of the sphere $x^2+y^2+z^2 = 4$ that lies above the cone $z = \sqrt{x^2 + y^2}$.

    (d) Use the surface area formula to find the area of the portion of the plane $au + bv + cw = d$ lying above (or below, it makes no difference) a domain R in the uv-plane. The answer will involve the area of R, area(R), which we cannot express as a number because R is not given explicitly.

27. (11/4) 16.6 # 7, 9, 15

28. (11/4) 16.6 # 21, 33 (just find limits for integrating with respect to $dy\,dz\,dx$, not all five other possibilities), 35 (just find limits for integrating with respect to $dy\,dx\,dz$, not all five other possibilities), 45(a)(b), 46(a)(b), 53

29. (11/9) 16.7 # 19, 25, 27, 16.8 # 11-14 (for 14, notice that the second equation says $\rho\sin(\phi) \le 1$, and recall what $\rho\sin(\phi)$ is), 17, 19, 20, 25

30. (11/9) 16.9 # 3, 9, 10, 12 (solving for u and v gives $u = x - y$ and $v = 3x + y$, and you find that the parallelogram becomes a square in the uv-plane, and the integral computes to 192), 14 (if you compute correctly, the uv-region works out to be the unit disk, and $x^2 - xy + y^2$ becomes $u^2 + v^2$)

31. (11/18, 11/21 or 11/22, your choice) 16.9 # 17(a)

32. (11/18, 11/21 or 11/22, your choice) 17.1 # 2 (what is the vector field like along each vertical line?), 4 (what is the vector field like along the straight lines $x - y = c$?), 11-14, 15-18, 25 (recall that the gradient must be perpendicular to the level curves $x^2 - y = c$), 29-32 (thinking about level curves may make it easier)

33. (11/18, 11/21 or 11/22, your choice) 17.2 # 3, 5, 11, 13, 33, 34 (for 33 and 34, dm represents the mass of a piece of the wire of length ds, so $dm = k\,ds$).

34. (11/18, 11/21 or 11/22, your choice) 17.2 # 17, 18, 19, 21, 17.3 # 1, 4, 5, 11, 13, 19, 27, 33