Mathematics 2423-001   Name (please print)

Exam II

October 17, 2011

Instructions: Give concise answers, but clearly indicate your reasoning. It is *not* expected that you will be able to answer all the questions, just do whatever you can in 50 minutes.

I.   Evaluate the following integrals.
(12)

  1. $\int \dfrac{\cos(\pi/x)}{x^2}\, dx$

  2. $\int \dfrac{x+1}{x^2+1}\, dx$

  3. $\int \dfrac{x^2}{\sqrt{1-x}}\, dx$

II.   If $f(x)$ is the slope of a trail at a distance x miles from the start of the trail, what does the integral
(3) $\displaystyle\int_3^5 f(x)\, dx$ represent?

III.   By substituting $u = \dfrac{t}{a}$, verify that $\displaystyle\int_a^{ab} \dfrac{1}{t}\, dt = \ln(b)$.
(4)

IV.   Write definite integrals to compute each of the following, but *do not* simplify or evaluate them.
(8)

  (a) The volume of the solid produced when the region bounded by $y = (x-2)^2$ and $y = 8x - 16$ is rotated about the line $y = -1$.

  (b) The volume of the solid produced when the region in part (a) is rotated about the line $x = -1$.

V.   (a) Calculate and simplify: $\dfrac{d}{dx} \ln(x + \sqrt{x^2 - 1})$
(11)

  (b) Simplify and calculate: $\dfrac{d}{dz} \ln\left(\sqrt{\dfrac{a^2 - z^2}{a^2 + z^2}} \right)$

  (c) Calculate the average value of $\dfrac{1}{1 + x^2}$ between $x = 0$ and $x = \sqrt{3}$.

VI.   Potpourri:
(13)

  1. Define what it means to say that a function f is *injective.*

  2. For an injective function f with domain A and range B, define the *inverse function g.*

  3. State the Intermediate Value Theorem.

  4. Show that for any integer $n \geq 2$, $\ln(n) < 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1}$.

  5. Draw a right triangle with hypotenuse of length 1 and a side of length x. Indicate the length of the third side and correctly label the interior angles as $\sin^{-1}(x)$ and $\cos^{-1}(x)$. Use the triangle to find $\cot(\sin^{-1}(x))$.