Mathematics 2423-001 Name (please print) ____________________

Final Exam

December 16, 2011

Instructions: Give concise answers, but clearly indicate your reasoning. It is *not* expected that you will be able to answer all the questions, just do whatever you can.

**I.** (3) Explain how we know that every continuous function has an antiderivative.

**II.** (6) For each of the following, write the partial fraction decomposition with unknown coefficients in the numerators, but do not go on to solve for the coefficients.

1. $\dfrac{1}{(x^2+x)^2}$

2. $\dfrac{1}{(x-1)(x^2+1)(x^4-1)}$

**III.** (6) (a) Briefly explain the idea of Simpson's Rule. Feel free to make use of a meaningful picture.

(b) Given the following fact:

If $P(x)$ is the parabola passing through the points $(x_0, y_0)$, $(x_1, y_1)$, $(x_2, y_2)$ with $h = x_1 - x_0 = x_2 - x_1$, then $\displaystyle\int_{x_0}^{x_2} P(x)\,dx = \frac{h}{3}(y_0 + 4y_1 + y_2)$.

obtain the formula in Simpson's Rule.

**IV.** (20) Find each of the following.

(a) $\displaystyle\int \frac{1}{(1+x^2)^2}\,dx$ (you will need the trig identities $\cos^2(\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$ and $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$).

(b) $\displaystyle\int \sin(\ln(x))\,dx$ (start by using the inverse substitution $x = e^u$, then integrate by parts twice)

(c) $\displaystyle\lim_{x\to 0^+} \sin(x)\ln(x)$

(d) $f(x)$, if $\displaystyle\int_0^x f(t)\,dt = xe^{2x} + \int_0^x e^{-t} f(t)\,dt$

(e) $\displaystyle\int \frac{1}{\sqrt{x^2+x}}\,dx$, given that $\displaystyle\int \frac{1}{\sqrt{u^2-a^2}}\,du = \ln|u + \sqrt{u^2-a^2}| + C$.

**V.** (9) This problem concerns the curve which is the portion of the graph $y = 3 + \frac{1}{2}\cosh(2x)$ between $x = 0$ and $x = 1$.

(a) Find $ds$ for this curve.

(b) Calculate the length of the curve.

(c) Write an integral whose value equals the surface area produced when the curve is rotated about the $x$-axis, but do not evaluate the integral.

**VI.** (4) Carry out integration by parts to reduce the evaluation of $\int \frac{x \arctan(x)}{(1+x^2)^2}\,dx$ to a problem of integrating a rational function, but do not continue on to integrate that rational function.

**VII.** (6) If $f(t)$ is continuous for $t \geq 0$, the *Laplace transform* of $f$ is the function $F(s)$ defined by $F(s) = \int_0^\infty f(t)\,e^{-st}\,dt$. Find $F(s)$ if $f(t) = e^{kt}$. Be sure to tell the domain of this $F(s)$.

**VIII.** (9) Recall that we defined $f'(a)$ by

$$f(a+h) = f(a) + f'(a)h + E(h)\ ,$$

where $\lim_{h\to 0} E(h)/h = 0$.

(a) Draw a picture showing the graph of a typical $f$, $a$, $a+h$, $f(a)$, $f(a+h)$, $f'(a)h$, and $E(h)$.

(b) Use the definition to find $f'$ if $f(x) = x^2$.

(c) Use integration by parts to show that $E(h) = \int_a^{a+h} (a+h-t)f''(t)\,dt$.

**IX.** (6) Let $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$ be a partition of the closed interval $[a,b]$.

(a) Define a *Riemann sum* for $f$ on the interval $[a,b]$, associated to this partition.

(b) Define $\int_a^b f(x)\,dx$

(c) For the function $f(x) = x^2$ and the interval $[-1,2]$, find the smallest Riemann sum associated to the partition $-1 < -1/2 < 1 < 2$.

**X.** (6) (a) State the Mean Value Theorem.

(b) Let $F(x) = \int_0^x f(t)\,dt$. Tell why $\int_a^b f(t)\,dt = F(b) - F(a)$. Then use the Mean Value Theorem to tell why $\int_a^b f(t)\,dt = f(c)(b-a)$ for some $c$ in the interval $(a,b)$ (this is the "Mean Value Theorem for Integrals").