Math 2423 homework

1. (due 8/26) Following the approach we used in class to calculate the rate of change of the sine function, calculate the rate of change of the cosine function as follows:

    1. Use the addition formula to expand $\cos(a+h)$.
    2. Rewrite the expansion as $\cos(a+h) = \cos(a) + (-\sin(a))h + E(h)$, where $E(h)$ is of the form something $\cdot \cos(a) +$ something $\cdot \sin(a)$.
    3. Use the trigonometric limits we discussed in class to verify that $\lim_{h\to 0} \frac{E(h)}{h} = 0$, thus verifying that $-\sin(a)$ is the rate of change of the cosine function at a.

2. (due 8/26) For the function $f(x) = x^2 + 7x$, verify that 3 is not the rate of change at 0 as follows:

    1. Write $(0+h)^2 + 7(0+h) = 0 + 3h + E(h)$ for an explicit $E(h)$.
    2. Calculate $\lim_{h\to 0} \frac{E(h)}{h}$, obtaining a nonzero value. Conclude that 3 is not the rate of change.

    Then, repeat the process for the value of m that *is* the rate of change, in which case $\lim_{h\to 0} \frac{E(h)}{h}$ is 0.
    Illustrate both cases with pictures that clarify what is going on geometrically.

3. (due 8/31) For $f(x) = x^3$ and $[a,b] = [0,2]$, find a number c that satisfies the conclusion of the MVT.

4. (due 8/31) Use our estimate of $E(h)$ in terms of f'' to give an estimate of the error if one uses linear approximation at $a = 0$ to estimate $\cos(0.2)$. Draw a picture of the linear approxmation, showing $E(h)$, and use your estimate of $|E(h)|$ to determine a range for the true value of $\cos(0.2)$.

5. (due 8/31) Use the MVT to verify that $|\sin(x) - \sin(y)| \leq |x - y|$ for all x and y.

6. (due 8/31) Use the MVT to verify that $\sqrt{1+x} < 1+\frac{x}{2}$ for all $x > 0$ (take $f(x) = \sqrt{1+x}$ and $[a,b] = [0,x]$).