I. (a) Find the differential of \( \sqrt{x} \).

(b) Use linear approximation to estimate \( \sqrt{3.996} \).

II. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 2 m higher than the bow of the boat. The rope is pulled at a rate of 2 m/sec.

(a) Draw and label a figure that illustrates this situation.

(b) How fast is the boat approaching the dock when it is 12 m from the dock?
III. In this problem, \( f(x) = \frac{2}{x^2} + \frac{1}{x^2} \), which can also be written as \( \frac{2}{x^2} (x + \frac{1}{2}) \). The first and second derivatives of \( f \) are \( f'(x) = -\frac{2}{x^2} - \frac{2}{x^3} \) and \( f''(x) = \frac{4}{x^3} + \frac{6}{x^4} \) (do not check these)

1. Find the root or roots of \( f \).

2. Find \( \lim_{x \to \infty} f(x) \).

3. Find \( \lim_{x \to 0^+} f(x) \) and \( \lim_{x \to 0^-} f(x) \).

4. Find the critical number or critical numbers of \( f \).

5. Find the inflection point or inflection points of \( f \).

IV. State the Mean Value Theorem, including its hypotheses.

(4)
V. (a) For the function \( \sqrt{x} \) on the interval \([0, 9]\), find a number \( c \) that satisfies the conclusion of the Mean Value Theorem.

(b) Use the Mean Value Theorem to verify the following fact: Suppose that \( f: [a, b] \to \mathbb{R} \) is continuous on \([a, b]\) and differentiable on \((a, b)\). If \( f'(x) < 0 \) for \( a < x < b \), then \( f(a) > f(b) \).

VI. A certain differentiable function \( f \) has domain all nonzero real numbers, and has the following properties:

(a) \( f(-2) = 1, \ f(3) = 4, \ f(5) = 2 \)

(b) \( f'(-2) = 0, \ f'(3) = 0 \).

(c) \( \lim_{x \to 0} f(x) = -\infty \).

(d) \( f''(x) < 0 \) for \(-2 < x < 0\) and for \( 0 < x < 5 \).

(e) \( f''(x) > 0 \) for \( x < -2 \) and \( 5 < x \).

(f) \( \lim_{x \to \infty} f(x) = 1 \).

Sketch a possible graph of \( f \), using all of the above information.
Let $f : [a, b] \to \mathbb{R}$ be a function which is continuous on $[a, b]$ and differentiable on $(a, b)$. Label each of the following statements either T for true or F for false.

_____ If $a$ and $b$ are roots of $f$, then there must exist a number $c$ between $a$ and $b$ for which $f'(c) = 0$.

_____ If $f'(x) > 0$ for $a < x < b$, then $f(a) < f(b)$.

_____ The Mean Value Theorem is a special case of Rolle’s Theorem.

_____ The Mean Value Theorem can be deduced from Rolle’s Theorem.

_____ If $f(a) < f(b)$, then there must exist a number $c$ between $a$ and $b$ for which $f'(c) > 0$.

_____ If $f''(x)$ changes sign at $c$, then $c$ is an inflection point of $f$.

_____ There must exist a number $c$ in the interval $[a, b]$ such that $f(c) \geq f(x)$ for all $x$ in $[a, b]$.

_____ If $f$ were not continuous on $[a, b]$, then it could still have an absolute maximum or absolute minimum value on $[a, b]$, but not both.

_____ If $c$ is an interior point of the interval and $f'(c) = 0$, then $f$ must have either a local maximum or a local minimum (or both) at $c$.

_____ If $f(x)$ equals the mass of the portion of a metal rod between 0 and $x$, then $f''(x)$ is the density function of the rod.