In this problem, \( f(x) = \frac{2}{x} + \frac{1}{x^2} \), which can also be written as \( \frac{2}{x^2}(x + \frac{1}{2}) \). The first and second derivatives of \( f \) are

\[
\frac{df}{dx} = -\frac{2}{x^2} - \frac{2}{x^3}
\]

and

\[
\frac{d^2f}{dx^2} = \frac{4}{x^3} + \frac{6}{x^4}
\]

1. Find the root or roots of \( f \).

2. Find \( \lim_{x \to \infty} f(x) \).

3. Find the critical number or critical numbers of \( f \).

4. Find the inflection point or inflection points of \( f \).

5. Find \( \lim_{x \to 0^+} f(x) \) and \( \lim_{x \to 0^-} f(x) \).

II.

(a) Find the differential of \( \sqrt{x} \).

(b) Use linear approximation to estimate \( \sqrt{8.994} \).
III. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 3 m higher than the bow of the boat. The rope is pulled at a rate of 3 m/sec.

(a) Draw and label a figure that illustrates this situation.

(b) How fast is the boat approaching the dock when it is 9 m from the dock?

IV. State the Mean Value Theorem, including its hypotheses.

(b) For the function $\sqrt{x}$ on the interval $[0, 4]$, find a number $c$ that satisfies the conclusion of the Mean Value Theorem.
V. Use the Mean Value Theorem to verify the following fact: Suppose that $f : [a, b] \to \mathbb{R}$ is continuous on $[a, b]$ and differentiable on $(a, b)$. If $f'(x) > 0$ for $a < x < b$, then $f(a) < f(b)$.

VI. A certain differentiable function $f$ has domain all real numbers except $x = -2$, and has the following properties:

(a) $f(-3) = -1$, $f(1) = 1$, $f(3) = 2$

(b) $f'(-3) = 0$, $f'(1) = 0$.

(c) $\lim_{x \to -2} f(x) = -\infty$.

(d) $f''(x) < 0$ for $x < -2$, for $-2 < x < 1$, and for $3 < x$.

(e) $f''(x) > 0$ for $1 < x < 3$.

(f) $\lim_{x \to \infty} f(x) = 3$.

Sketch a possible graph of $f$, using all of the above information.
Let \( f : [a, b] \to \mathbb{R} \) be a function which is continuous on \([a, b]\) and differentiable on \((a, b)\). Label each of the following statements either \(T\) for true or \(F\) for false.

- If \( f'(x) < 0 \) for \( a < x < b \), then \( f(a) < f(b) \).
- If \( a \) and \( b \) are roots of \( f \), then there must exist a number \( c \) between \( a \) and \( b \) for which \( f'(c) = 0 \).
- The Mean Value Theorem can be deduced from Rolle’s Theorem.
- The Mean Value Theorem is a special case of Rolle’s Theorem.
- If \( f(a) < f(b) \), then there must exist a number \( c \) between \( a \) and \( b \) for which \( f'(c) > 0 \).
- There must exist a number \( c \) in the interval \([a, b]\) such that \( f(c) \geq f(x) \) for all \( x \) in \([a, b]\).
- If \( f \) were not continuous on \([a, b]\), then it could still have an absolute maximum or absolute minimum value on \([a, b]\), but not both.
- If \( c \) is an interior point of the interval and \( f'(c) = 0 \), then \( f \) must have either a local maximum or a local minimum (or both) at \( c \).
- If \( f''(x) \) changes sign at \( c \), then \( c \) is an inflection point of \( f \).
- If \( f(x) \) equals the mass of the portion of a metal rod between 0 and \( x \), then \( f'(x) \) is the density function of the rod.