Mathematics 1823-030
Examination II Form B
October 19, 2009

Name (please print) ____________________

Student Number ______________

(1) **Discussion Section** (circle day and time):

Th 9:00 Th 1:30 Th 3:00 F 8:30 F 9:30 F 2:30

I. (25) Calculate each of the following. When calculating derivatives, use the algebraic rules, the Chain Rule, and/or implicit differentiation (i. e. do not use the definition of the derivative as a limit). Do not perform simplifications to the answer unless instructed to do so.

(i) $\dfrac{dy}{dx}$ if $y = \dfrac{\cos(x^2)}{\cos^2(x)}$

$$\frac{d}{dx}\frac{\cos(x^2)}{\cos^2(x)} = \frac{\cos^2(x)\,(-\sin(x^2)\,2x) - \cos(x^2)(-2\cos(x)\sin(x))}{\cos^4(x)} = \frac{-2x\cos(x)\,\sin(x^2) + 2\sin(x)\cos(x^2)}{\cos^3(x)}$$

(final simplification not necessary)

(ii) $\dfrac{d^2y}{dx^2}$ if $y = \tan(x)$

$$\frac{d^2(\tan(x))}{dx^2} = \frac{d}{dx}(\sec^2(x)) = 2\sec(x)\,(\sec(x)\tan(x)) = 2\sec^2(x)\tan(x)$$

(iii) $\dfrac{dw}{dt}$ if $w = \sqrt{\dfrac{t+1}{t-1}}$. Simplify this answer.

$$\frac{d}{dt}\sqrt{\frac{t+1}{t-1}} = \frac{1}{2\sqrt{\dfrac{t+1}{t-1}}}\;\frac{(t-1)\cdot 1 - 1\cdot(t+1)}{(t-1)^2} = \frac{-2}{2(t-1)^2\sqrt{\dfrac{t+1}{t-1}}} = \frac{-1}{(t-1)^{3/2}\sqrt{t+1}}$$

(iv) An equation for the tangent line to the curve $y = (x^2-1)^{1{,}000}$ at the point $(\sqrt{2}, 1)$. You need not simplify the answer.
$\dfrac{dy}{dx} = 1{,}000\cdot(x^2-1)^{999}\cdot 2x$, so the slope of the tangent line is $\dfrac{dy}{dx}\Big|_{x=\sqrt{2}} = 1{,}000(1)\cdot 2\sqrt{2} = 2{,}000\sqrt{2}$.
Therefore an equation for the tangent line is

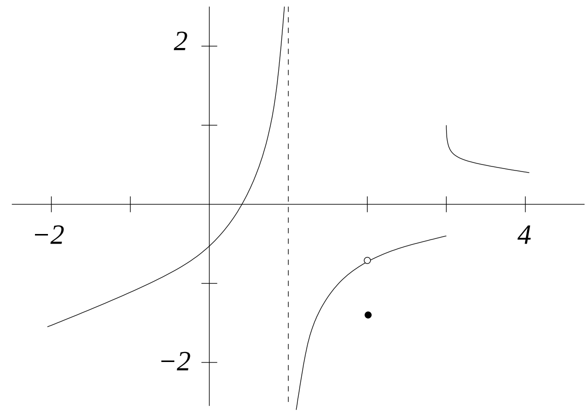
$$y - 1 = 2{,}000\sqrt{2}(x - \sqrt{2})$$
$$y = 2{,}000\sqrt{2}x - 3{,}999$$

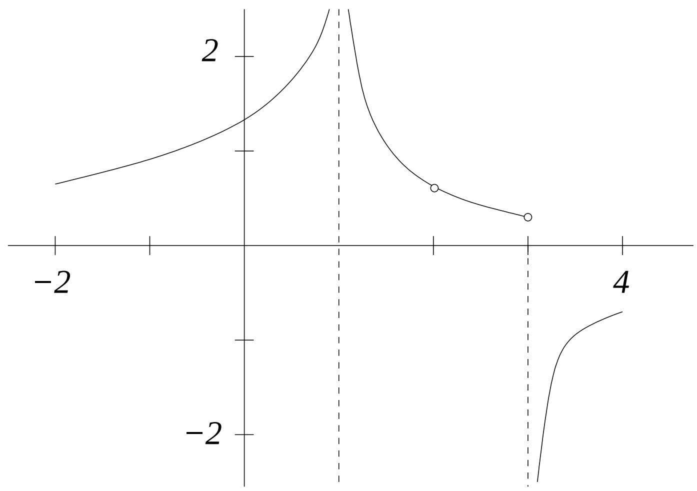
(final simplification not necessary)

(v) $\dfrac{dy}{dx}$ if $x^2\sin(y) = y^3$

$$x^2\cos(y)\frac{dy}{dx} + 2x\sin(y) = 3y^2\frac{dy}{dx}$$
$$(x^2\cos(y) - 3y^2)\frac{dy}{dx} = -2x\sin(y)$$
$$\frac{dy}{dx} = \frac{-2x\sin(y)}{x^2\cos(y) - 3y^2}$$

II. The figure to the right shows the graph of a certain function $f\colon [-2,4] \to \mathbb{R}$. On the coordinate system shown
(6) below, sketch a graph of the derivative $f'(x)$. The values of $f'(x)$ need not be precise, but should accurately reflect the behavior of $f(x)$. Indicate clearly any points where $f'(x)$ is underfined.





III. Find the limit $\lim\limits_{\theta\to 0} \dfrac{\sin(7\theta)}{\sin(5\theta)}$ (not by plotting points or by using l'Hôpital's rule).
(4)

$$\lim_{\theta\to 0} \frac{\sin(7\theta)}{\sin(5\theta)} = \lim_{\theta\to 0} \frac{\frac{\sin(7\theta)}{7\theta}\,7\theta}{\frac{\sin(5\theta)}{5\theta}\,5\theta} = \lim_{\theta\to 0} \frac{\frac{\sin(7\theta)}{7\theta}}{\frac{\sin(5\theta)}{5\theta}}\,\frac{7}{5}\,\frac{\theta}{\theta} = \frac{1}{1}\,\frac{7}{5}\,1 = \frac{7}{5}$$

IV. (3) State the precise, formal (i. e. using ϵ and δ) definition of: $\lim_{x\to 3\pi/4} \sec(x) = -\sqrt{2}$.

For every $\epsilon > 0$, there exists $\delta > 0$ such that if $0 < |x - 3\pi/4| < \delta$, then $|\sec(x) + \sqrt{2}| < \epsilon$.

V. (5) Define what it means to say that a function f is *continuous at* x_0. State the Intermediate Value Theorem.

f is *continuous* at x_0 when f is defined at x_0, $\lim_{x\to x_0} f(x)$ exists, and $\lim_{x\to x_0} f(x) = f(x_0)$.

The Intermediate Value Theorem says that if f is a continous function on the closed interval $[a, b]$, and N is any number between $f(a)$ and $f(b)$, then there exists a c between a and b such that $f(c) = N$.

VI. (6) Let f be a function which is differentiable at $x = a$. Label each of the following statements either T for true or F for false.

__T__ The limit $\lim_{w\to a} \frac{f(w) - f(a)}{w - a}$ must exist.

__T__ f must be continuous at $x = a$.

__F__ f must be differentiable on any open interval that contains a.

__F__ $f'(a)$ might be ∞ or $-\infty$.

VII. (4) In the blank to the left of each of the following two questions, write the letter of the best response.

1. __C__ What type of mathematical object is $\frac{d^2y}{dx^2}$?

A) set B) equation C) function D) codomain E) theorem F) number

2. __D__ The pair $(4!, 0!)$ equals

A) $(4, 1)$ B) $(12, 1)$ C) $(20, 1)$ D) $(24, 1)$ E) $(32, 1)$ F) $(120, 1)$
G) $(4, 0)$ H) $(12, 0)$ I) $(20, 0)$ J) $(24, 0)$ K) $(32, 0)$ L) $(120, 0)$

VIII. (6) The table to the right shows the values of the functions f, g, f', and g' at the x-values 1, 2, 3, and 4. For example, $f(4) = 4$ and $f'(4) = 1$. Write the value of each of the following:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	3	1	4
2	2	4	1	2
3	1	2	4	2
4	4	1	4	2

$(g \cdot f)'(3) =$ __10__ $(g \circ f)'(1) =$ __6__ $(f/g)'(4) =$ __$-\frac{1}{4}$__ $(f \circ f)(3) =$ __2__