

(1) **Discussion Section** (circle day and time):

Th 9:00   Th 1:30   Th 3:00   F 8:30   F 9:30   F 2:30

I. Calculate each of the following. When calculating derivatives, use the algebraic rules, the Chain Rule, (25) and/or implicit differentiation (i. e. do not use the definition of the derivative as a limit). Do not perform simplifications to the answer unless instructed to do so.

(i)  $\frac{dy}{dx}$  if  $y = \frac{\cos(x^2)}{\cos^2(x)}$

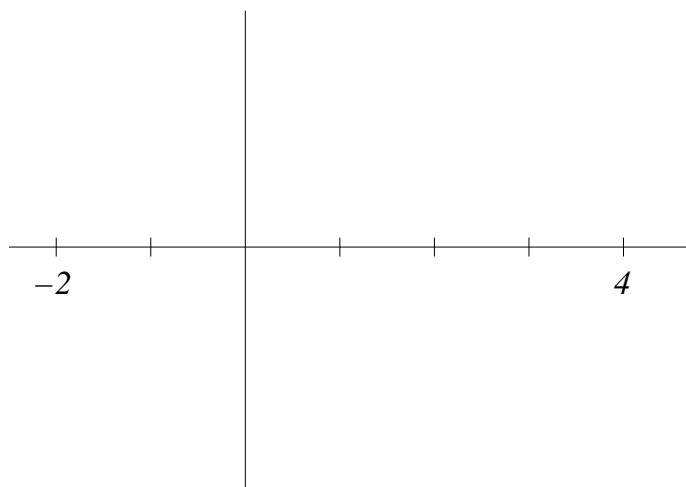
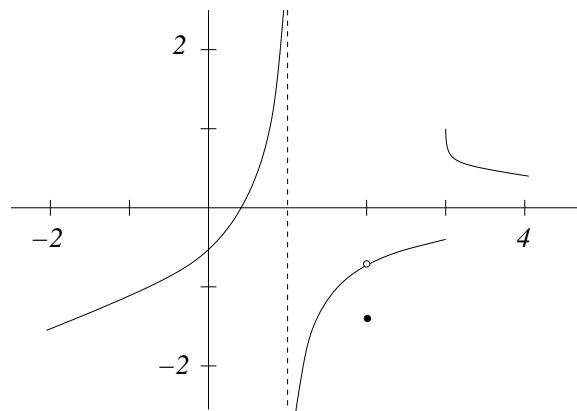
(ii)  $\frac{d^2y}{dx^2}$  if  $y = \tan(x)$

(iii)  $\frac{dw}{dt}$  if  $w = \sqrt{\frac{t+1}{t-1}}$ . Simplify this answer.

(iv) An equation for the tangent line to the curve  $y = (x^2 - 1)^{1,000}$  at the point  $(\sqrt{2}, 1)$ . You need not simplify the answer.

(v)  $\frac{dy}{dx}$  if  $x^2 \sin(y) = y^3$

- II.** The figure to the right shows the graph of a certain function  $f: [-2, 4] \rightarrow \mathbb{R}$ . On the coordinate system shown below, sketch a graph of the derivative  $f'(x)$ . The values of  $f'(x)$  need not be precise, but should accurately reflect the behavior of  $f(x)$ . Indicate clearly any points where  $f'(x)$  is undefined.



- III.** Find the limit  $\lim_{\theta \rightarrow 0} \frac{\sin(7\theta)}{\sin(5\theta)}$  (not by plotting points or by using l'Hôpital's rule).

(4)

IV. State the precise, formal (i. e. using  $\epsilon$  and  $\delta$ ) definition of:  $\lim_{x \rightarrow 3\pi/4} \sec(x) = -\sqrt{2}$ .  
(3)

V. Define what it means to say that a function  $f$  is *continuous at*  $x_0$ . State the Intermediate Value Theorem.  
(5)

VI. Let  $f$  be a function which is differentiable at  $x = a$ . Label each of the following statements either  $T$  for true or  $F$  for false.  
(6)

\_\_\_\_\_ The limit  $\lim_{w \rightarrow a} \frac{f(w) - f(a)}{w - a}$  must exist.

\_\_\_\_\_  $f$  must be continuous at  $x = a$ .

\_\_\_\_\_  $f$  must be differentiable on any open interval that contains  $a$ .

\_\_\_\_\_  $f'(a)$  might be  $\infty$  or  $-\infty$ .

VII. In the blank to the left of each of the following two questions, write the letter of the best response.  
(4)

1. \_\_\_\_\_ What type of mathematical object is  $\frac{d^2y}{dx^2}$ ?

A) set      B) equation      C) function      D) codomain      E) theorem      F) number

2. \_\_\_\_\_ The pair  $(4!, 0!)$  equals

A) (4, 1)      B) (12, 1)      C) (20, 1)      D) (24, 1)      E) (32, 1)      F) (120, 1)  
G) (4, 0)      H) (12, 0)      I) (20, 0)      J) (24, 0)      K) (32, 0)      L) (120, 0)

VIII. The table to the right shows the values of the functions  $f$ ,  $g$ ,  $f'$ , and  $g'$  at the  $x$ -values 1, 2, 3, and 4. For example,  $f(4) = 4$  and  $f'(4) = 1$ . Write the value of each of the following:  
(6)

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	3	1	4
2	2	4	1	2
3	1	2	4	2
4	4	1	4	2

$(g \cdot f)'(3) =$  \_\_\_\_\_       $(g \circ f)'(1) =$  \_\_\_\_\_       $(f/g)'(4) =$  \_\_\_\_\_       $(f \circ f)(3) =$  \_\_\_\_\_