Mathematics 1823-030
Examination I Form A
September 21, 2009

Name (please print) ______________________

Student Number ______________

(1) **Discussion Section** (circle day and time):

Th 9:00 Th 1:30 Th 3:00 F 8:30 F 9:30 F 2:30

I. (4) The table to the right shows the values of two functions f and g at the x-values 0, 1, 2, 3, and 4. For example, $f(1) = 3$ and $g(1) = 0$. Write the value of each of the following:

x	0	1	2	3	4
$f(x)$	2	3	0	4	1
$g(x)$	4	0	3	2	1

$(g \circ f)(3) =$ __1__ $(f \circ g)(3) =$ __0__ $(f \cdot f)(3) =$ __16__ $(f \circ f)(3) =$ __1__

II. (12) In the blank to the left of each of the following questions, write the letter of the best response.

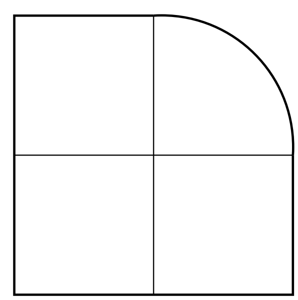
1. __A__ Let $f\colon \mathbb{R} \to \mathbb{R}$ (i. e. let f be a function from the real numbers to the real numbers). What type of mathematical object is the graph of f?

A) set B) function C) equation D) codomain E) number F) velocity

2. __E__ What type of mathematical object is $\lim_{x \to 2} \sin^3(x)$?

A) set B) function C) equation D) codomain E) number F) velocity

The next two questions refer to the figure to the right. It shows a window consisting of four panes, three of which are squares and one of which is a quarter of a disk. The width of the entire window is x.



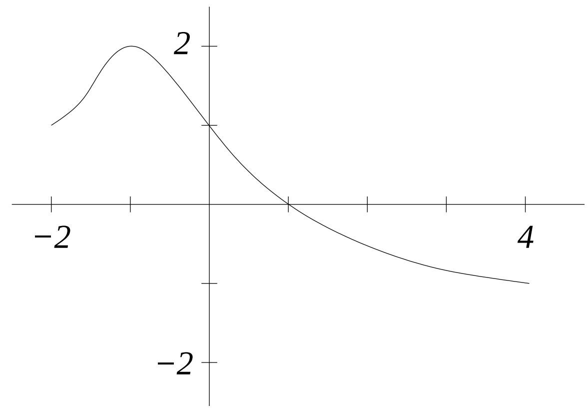
3. __F__ Which of the following is an expression for the *perimeter* of the window as a function of x?

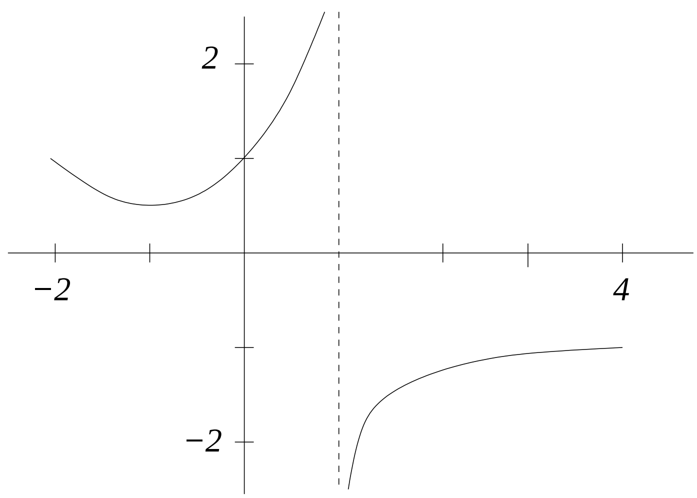
A) $2x + \pi x/2$ B) $2x + \pi x/4$ C) $2x + \pi x^2/2$ D) $2x + \pi x^2/4$ E) $3x + \pi x/2$ F) $3x + \pi x/4$
G) $3x + \pi x^2/2$ H) $3x + \pi x^2/4$ I) $6x + \pi x/2$ J) $6x + \pi x/4$ K) $6x + \pi x^2/2$ L) $6x + \pi x^2/4$

4. __C__ Which of the following is an expression for the *area* of the window as a function of x?

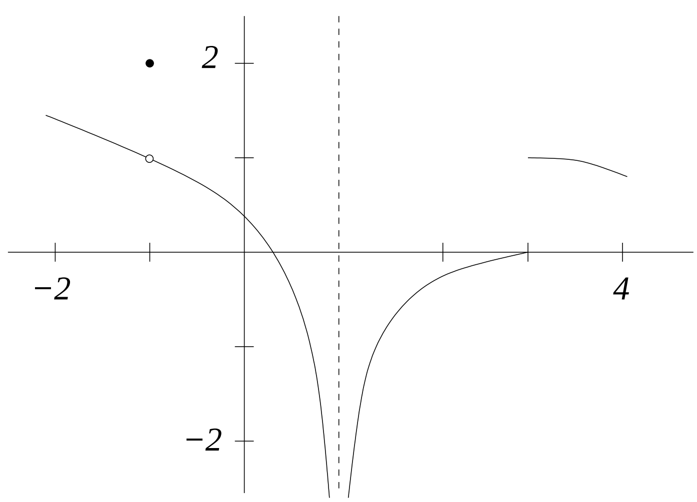
A) $3x^2 + \pi x^2/16$ B) $3x^2/2 + \pi x^2/16$ C) $3x^2/4 + \pi x^2/16$ D) $3x^2 + \pi x^2/4$ E) $3x^2/2 + \pi x^2/4$ F) $3x^2/4 + \pi x^2/4$
G) $3x^2 + \pi x^2/2$ H) $3x^2/2 + \pi x^2/2$ I) $3x^2/4 + \pi x^2/2$ J) $3x^2 + \pi x^2$ K) $3x^2/2 + \pi x^2$ L) $3x^2/4 + \pi x^2$

III. (4) The figure to the right shows the graph of a certain function $f\colon [-2,4] \to \mathbb{R}$. On the coordinate system shown below, sketch the graph of the reciprocal function $\dfrac{1}{f(x)}$. Make the y-values reasonably accurate, based on the values of $f(x)$.



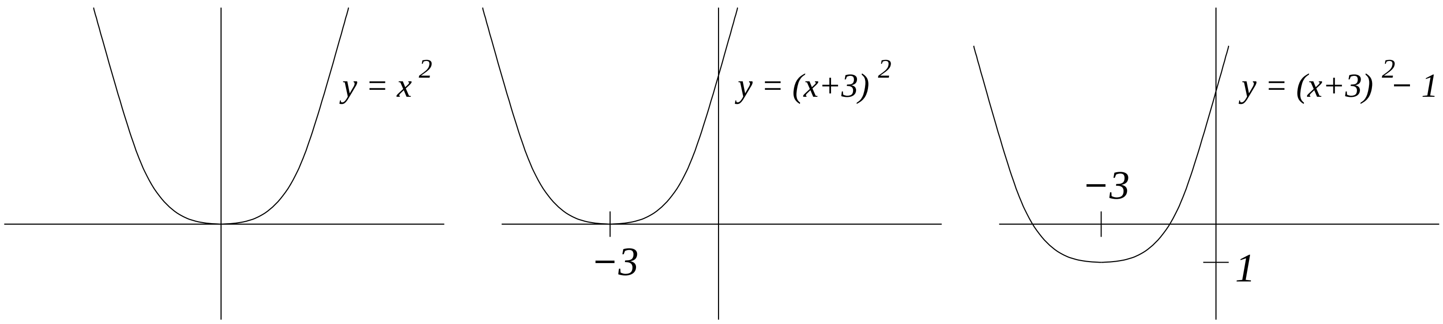


IV. (5) On the coordinate system shown below, sketch the graph of a function f that satisfies all of the following: $\lim\limits_{x\to -1} f(x)$ exists but f is not continuous at $x=-1$, $\lim\limits_{x\to 1} f(x) = -\infty$, $\lim\limits_{x\to 3^-} f(x) = 0$, and $\lim\limits_{x\to 3^+} f(x) = 1$.

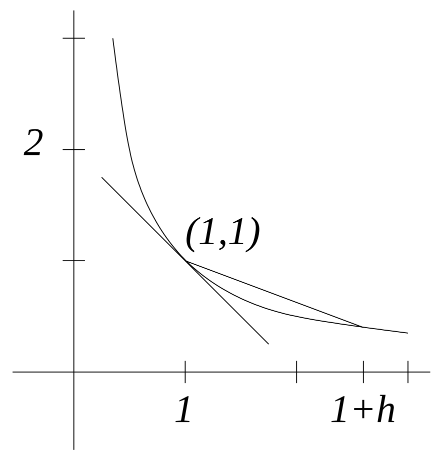


V. (4) Use completing the square and translation to graph the function $y = x^2 + 6x + 8$.

$$x^2 + 6x + 8 = (x^2 + 6x + 9) + 8 - 9 = (x + 3)^2 - 1$$



VI. (5) The figure to the right shows a portion of the graph of the function $f(x) = \dfrac{1}{x}$. It also shows the tangent line at the point $(1, 1)$, and a typical secant line.



(a) One of the endpoints of the secant line is $(1, 1)$. Give the coordinates of the other endpoint in terms of h.

$$\left(1 + h, \frac{1}{1 + h}\right)$$

(b) Calculate the slope of the secant line as a function m_h of h.

$$m_h = \frac{\frac{1}{1+h} - 1}{(1+h) - 1} = \frac{\frac{1}{1+h} - \frac{1+h}{1+h}}{h} = \frac{-h}{h(1+h)}$$

(c) Evaluate the limit $\lim_{h \to 0} m_h$ to find the slope m_{tan} of the tangent line at $(1, 1)$.

$$m_{tan} = \lim_{h\to 0} m_h = \lim_{h\to 0} \frac{-h}{h(1+h)} = \lim_{h\to 0} \frac{-h}{h} \frac{1}{1+h} = (-1)\,\frac{1}{1+0} = -1$$

VII. Define what it means to say that a function f is *continuous at* x_0. State the Intermediate Value Theorem.
(5)

f is *continuous* at x_0 when f is defined at x_0, $\lim_{x \to x_0} f(x)$ exists, and $\lim_{x \to x_0} f(x) = f(x_0)$.

The Intermediate Value Theorem says that if f is a continous function on the closed interval $[a, b]$, and N is any number between $f(a)$ and $f(b)$, then there exists a c between a and b such that $f(c) = N$.

VIII. State the precise, formal (i. e. using ϵ and δ) definition of: $\lim_{x \to \pi/4} \cos(x) = 1/\sqrt{2}$.
(3)

For every $\epsilon > 0$, there exists $\delta > 0$ such that if $0 < |x - \pi/4| < \delta$, then $|\cos(x) - 1/\sqrt{2}| < \epsilon$.

IX. Determine the following limits (not by plugging in values, and not by using l'Hôpital's rule).
(8)

1. $\lim_{h \to 2} \dfrac{\sqrt{h+2} - 2}{h - 2}$

$$
[\lim_{h \to 1} \frac{\sqrt{h+2} - 2}{h - 2} = \lim_{h \to 2} \frac{\sqrt{h+2} - 2}{h - 2} \, \frac{\sqrt{h+2} + 2}{\sqrt{h+2} + 2} = \lim_{h \to 2} \frac{(h+2) - 4}{h - 2} \, \frac{1}{\sqrt{h+2} + 2}
$$
$$
= \lim_{h \to 2} \frac{h - 2}{h - 2} \, \frac{1}{\sqrt{h+2} + 2} = 1 \cdot \frac{1}{\sqrt{2+2} + 2} = \frac{1}{4}
$$

2. $\lim_{x \to -3^+} \dfrac{1 + x}{3 + x}$

For x near -3 and a little larger than -3, the numerator is close to -2 and the denominator is a very small positive number, so the quotient is a large negative number. Therefore

$$
\lim_{x \to -3^+} \frac{1 + x}{3 + x} = -\infty
$$