Mathematics 1823-030
Examination I Form A
September 21, 2009

Name (please print) _____

Student Number _____

(1) **Discussion Section** (circle day and time):
Th 9:00 Th 1:30 Th 3:00 F 8:30 F 9:30 F 2:30

I. (4) The table to the right shows the values of two functions f and g at the x-values 0, 1, 2, 3, and 4. For example, $f(1) = 3$ and $g(1) = 0$. Write the value of each of the following:

x	0	1	2	3	4
$f(x)$	2	3	0	4	1
$g(x)$	4	0	3	2	1

$(g \circ f)(3) =$ _____ $(f \circ g)(3) =$ _____ $(f \cdot f)(3) =$ _____ $(f \circ f)(3) =$ _____

II. (12) In the blank to the left of each of the following questions, write the letter of the best response.

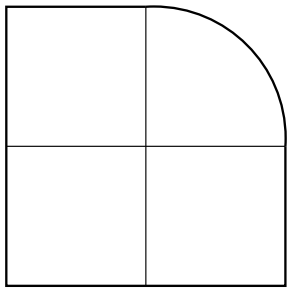
1. _____ Let $f\colon \mathbb{R} \to \mathbb{R}$ (i. e. let f be a function from the real numbers to the real numbers). What type of mathematical object is the graph of f?

A) set B) function C) equation D) codomain E) number F) velocity

2. _____ What type of mathematical object is $\lim_{x \to 2} \sin^3(x)$?

A) set B) function C) equation D) codomain E) number F) velocity

The next two questions refer to the figure to the right. It shows a window consisting of four panes, three of which are squares and one of which is a quarter of a disk. The width of the entire window is x.



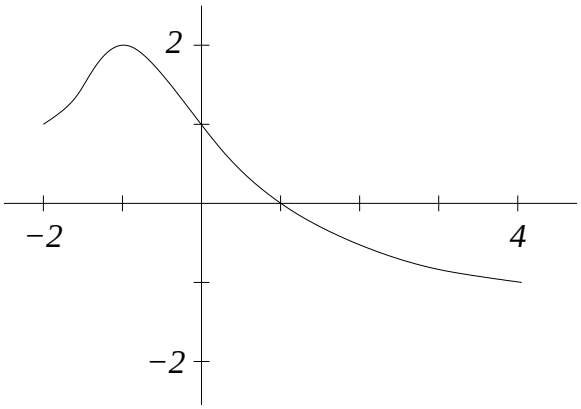
3. _____ Which of the following is an expression for the *perimeter* of the window as a function of x?

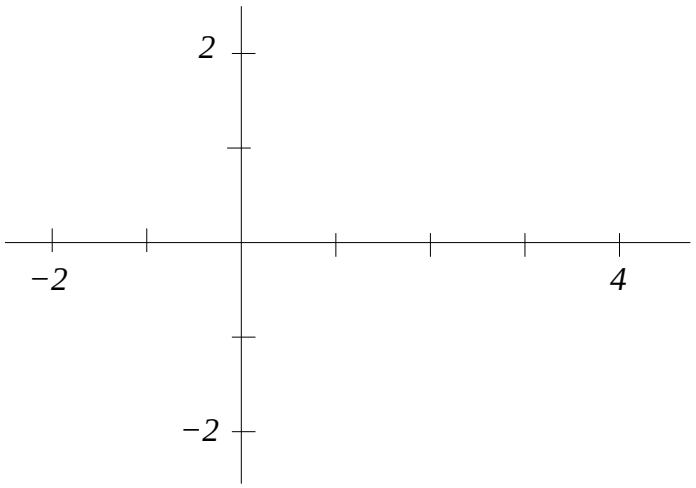
A) $2x + \pi x/2$ B) $2x + \pi x/4$ C) $2x + \pi x^2/2$ D) $2x + \pi x^2/4$ E) $3x + \pi x/2$ F) $3x + \pi x/4$
G) $3x + \pi x^2/2$ H) $3x + \pi x^2/4$ I) $6x + \pi x/2$ J) $6x + \pi x/4$ K) $6x + \pi x^2/2$ L) $6x + \pi x^2/4$

4. _____ Which of the following is an expression for the *area* of the window as a function of x?

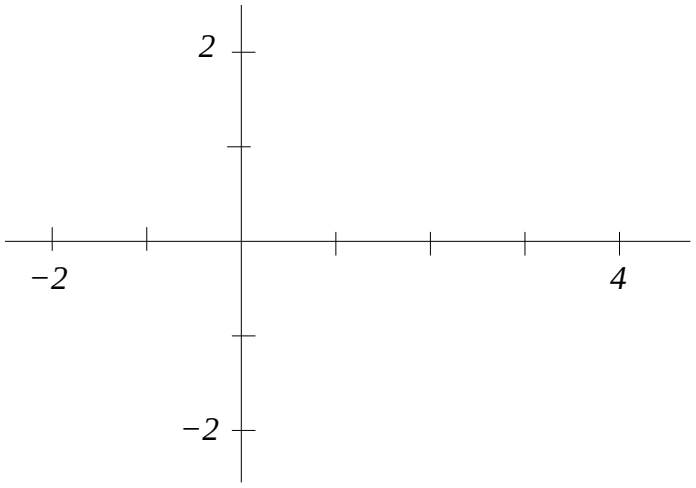
A) $3x^2 + \pi x^2/16$ B) $3x^2/2 + \pi x^2/16$ C) $3x^2/4 + \pi x^2/16$ D) $3x^2 + \pi x^2/4$ E) $3x^2/2 + \pi x^2/4$ F) $3x^2/4 + \pi x^2/4$
G) $3x^2 + \pi x^2/2$ H) $3x^2/2 + \pi x^2/2$ I) $3x^2/4 + \pi x^2/2$ J) $3x^2 + \pi x^2$ K) $3x^2/2 + \pi x^2$ L) $3x^2/4 + \pi x^2$

III. (4) The figure to the right shows the graph of a certain function $f\colon [-2,4] \to \mathbb{R}$. On the coordinate system shown below, sketch the graph of the reciprocal function $\dfrac{1}{f(x)}$. Make the y-values reasonably accurate, based on the values of $f(x)$.



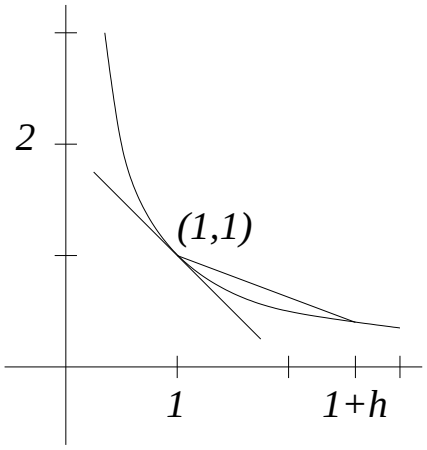


IV. (5) On the coordinate system shown below, sketch the graph of a function f that satisfies all of the following: $\lim\limits_{x\to -1} f(x)$ exists but f is not continuous at $x=-1$, $\lim\limits_{x\to 1} f(x) = -\infty$, $\lim\limits_{x\to 3^-} f(x) = 0$, and $\lim\limits_{x\to 3^+} f(x) = 1$.



V. (4) Use completing the square and translation to graph the function $y = x^2 + 6x + 8$.

VI. (5) The figure to the right shows a portion of the graph of the function $f(x) = \dfrac{1}{x}$. It also shows the tangent line at the point $(1, 1)$, and a typical secant line.



(a) One of the endpoints of the secant line is $(1, 1)$. Give the coordinates of the other endpoint in terms of h.

(b) Calculate the slope of the secant line as a function m_h of h.

(c) Evaluate the limit $\lim_{h \to 0} m_h$ to find the slope m_{tan} of the tangent line at $(1, 1)$.

VII. Define what it means to say that a function f is *continuous at* x_0. State the Intermediate Value Theorem. (5)

VIII. State the precise, formal (i. e. using ϵ and δ) definition of: $\lim_{x\to\pi/4} \cos(x) = 1/\sqrt{2}$. (3)

IX. Determine the following limits (not by plugging in values, and not by using l'Hôpital's rule). (8)

1. $\lim_{h\to 2} \dfrac{\sqrt{h+2}-2}{h-2}$

2. $\lim_{x\to -3^+} \dfrac{1+x}{3+x}$