Mathematics 1823-030
Final Examination Form B
December 15, 2009

Name (please print) ____________________

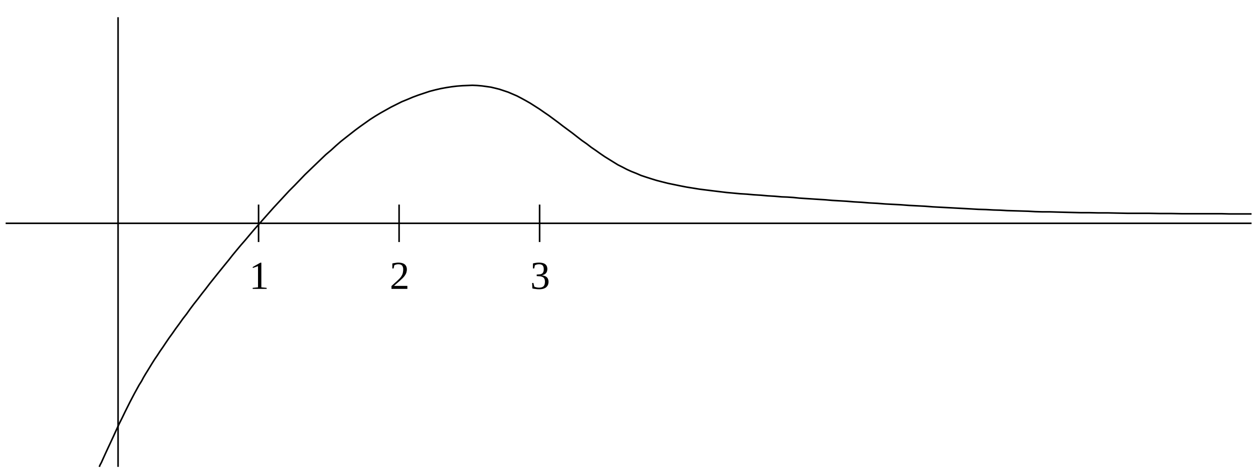
Student Number ______________

I. (6) The Law of Cosines formula is $c^2 = a^2 + b^2 - 2ab\cos(\theta)$. Let T be a triangle with two sides of lengths $2\sqrt{2}$ meters and 3 meters, and suppose that the angle θ between them is increasing at a rate of $\sqrt{5}$ radians per second. Sketch and label the triangle, and use the Law of Cosines to find the rate of change of the length of the third side when $\theta = \pi/4$.

II. (4) The graph below shows a function f having a root at $x = 1$.

(a) Suppose that Newton's method is applied starting the iteration with $x_1 = 2$. On the graph below, indicate where x_2, x_3, and x_4 would be, drawing appropriate tangent lines to clarify the relation of each iterate with the previous one.

(b) Similarly, starting over with $x_1 = 3$ on the same graph below, indicate what x_2, x_3, and x_4 would be in that case.



III. (7) (a) Write the precise (i. e. epsilon-delta) definition of $\lim_{x \to 2} 3x + 4 = 10$.

(b) Use the precise definition to verify that $\lim_{x \to 2} 3x + 4 = 10$.

IV. (5) The Mean Value Theorem implies that if $g\colon \mathbb{R} \to \mathbb{R}$ is a differentiable function and $g'(x) = 0$ for all x, then g is constant (do not verify this). Apply this fact to verify that if $f\colon \mathbb{R} \to \mathbb{R}$ is a differentiable function and F_1 and F_2 are antiderivatives of f, then there exists some constant C such that $F_1(x) = F_2(x) + C$ for all x.

V. (7) A farmer wants to fence an area of 600 square meters in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can this be done so as to minimize the cost of the fence?

VI. (6) Calculate each of the following limits:

1. $\lim\limits_{x\to 0} \dfrac{\sin(5x)}{x}$

2. $\lim\limits_{x\to\infty} \dfrac{x}{\sqrt{x^2+x}}$

3. $\lim\limits_{x\to-\infty} \dfrac{x}{\sqrt{x^2+x}}$

VII. (4) A certain differentiable function $f(x)$ satisfies $f(1) = 3$ and $f'(x) > 4$ for all x. Use the Mean Value Theorem to verify that $f(5) > 19$.

VIII. (5) Calculate $\dfrac{dy}{dx}$ if $\sin(y) = \sqrt{x^2+y^2}$.

IX. (12) Find each of the following:

1. The most general form for an antiderivative of $\sec^2(x) - 8x^3 + \sqrt{x}$ on the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

2. The most general form for $f(x)$ if $f''(x) = 12x - 4$.

3. $f(x)$, if $f'(x) = 6x^2 - 2$ and $f(\pi) = 2\pi^3$.

4. The differential of the function $\csc^2(2x)$.

X. (6) Let $f\colon \mathbb{R} \to \mathbb{R}$ (i. e. let f be a function from the real numbers to the real numbers). In the blank to the left of each of the following questions, write the letter of the best response.

1. _______ What type of mathematical object is the graph of f?

   A) function B) equation C) codomain D) number E) operation F) set

2. _______ If it exists, what type of mathematical object is $\lim_{x\to 2} f(x)$?

   A) function B) equation C) codomain D) number E) operation F) set

3. _______ If it exists, what type of mathematical object is $\lim_{x\to \infty} f(x)$?

   A) function B) equation C) codomain D) number E) operation F) set

4. _______ If it exists, what type of mathematical object is an antiderivative of f?

   A) function B) equation C) codomain D) number E) operation F) set

5. _______ What type of mathematical object is $f \circ f$?

   A) function B) equation C) codomain D) number E) operation F) set

6. _______ What type of mathematical object is composition of functions?

   A) function B) equation C) codomain D) number E) operation F) set

XI. (6) The table to the right shows the values of the functions f, g, f', and g' at the x-values 1, 2, 3, and 4. For example, $f(3) = 3$ and $f'(3) = 2$. Write the value of each of the following:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	4	1	2
2	2	3	1	4
3	3	2	4	1
4	4	1	2	2

$(g \cdot f)'(3) =$ _______ $(g \circ f)'(1) =$ _______ $(f/g)'(3) =$ _______ $(g \circ g)'(4) =$ _______

XII. (12) Let $g\colon \mathbb{R} \to \mathbb{R}$ be a function, not necessarily differentiable or continuous. Label each of the following statements either T for true or F for false.

_______ If g is continuous at a, then $\lim_{x \to a} g(x)$ must exist.

_______ If $\lim_{x \to a} g(x)$ exists, then g must be continuous at a.

_______ If f is continuous on a closed interval $[a, b]$, and N is any value between a and b, then there must exist c between $f(a)$ and $f(b)$ such that $f(c) = N$.

_______ The Extreme Value Theorem applies only to differentiable functions.

_______ If g is continuous at a, then it must be differentiable at a.

_______ If g is differentiable at a, then it must be continuous at a.

_______ Every function of the form $\frac{1}{x} + C$ for some value of C is an antiderivative of the function $-\frac{1}{x^2}$.

_______ Every antiderivative of the function $-\frac{1}{x^2}$ is of the form $\frac{1}{x} + C$ for some value of C.

_______ Both of the functions $\cos^2(x)$ and $\cos^2(x) + 5$ are antiderivatives of $-2\sin(x)$.

_______ If g is differentiable and G is an antiderivative of g, then g' tells the concavity of G.

_______ If g is differentiable and G is an antiderivative of g, then g' tells where G is increasing or decreasing.

_______ Newton's method can be applied to find approximations to solutions to equations, as well as to find approximations to roots.