I. The Mean Value Theorem implies that if \( g: \mathbb{R} \to \mathbb{R} \) is a differentiable function and \( g'(x) = 0 \) for all \( x \), then \( g \) is constant (do not verify this). Apply this fact to verify that if \( f: \mathbb{R} \to \mathbb{R} \) is a differentiable function and \( F_1 \) and \( F_2 \) are antiderivatives of \( f \), then there exists some constant \( C \) such that \( F_1(x) = F_2(x) + C \) for all \( x \).

II. (a) Write the precise (i.e. epsilon-delta) definition of \( \lim_{x \to 3} 2x + 5 = 11 \).

(b) Use the precise definition to verify that \( \lim_{x \to 3} 2x + 5 = 11 \).

III. A farmer wants to fence an area of 150 square meters in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can this be done so as to minimize the cost of the fence?
The graph below shows a function $f$ having a root at $x = 1$.

(a) Suppose that Newton’s method is applied starting the iteration with $x_1 = 2$. On the graph below, indicate where $x_2$, $x_3$, and $x_4$ would be, drawing appropriate tangent lines to clarify the relation of each iterate with the previous one.

(b) Similarly, starting over with $x_1 = 3$ on the same graph below, indicate what $x_2$, $x_3$, and $x_4$ would be in that case.

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The Law of Cosines formula is $c^2 = a^2 + b^2 - 2ab \cos(\theta)$. Let $T$ be a triangle with two sides of lengths $2\sqrt{2}$ meters and 3 meters, and suppose that the angle $\theta$ between them is increasing at a rate of $\sqrt{5}$ radians per second. Sketch and label the triangle, and use the Law of Cosines to find the rate of change of the length of the third side when $\theta = \pi/4$. 

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VI. A certain differentiable function \( f(x) \) satisfies \( f(1) = 3 \) and \( f'(x) > 5 \) for all \( x \). Use the Mean Value Theorem to verify that \( f(4) > 18 \).

VII. Calculate \( \frac{dy}{dx} \) if \( \sqrt{x^2 + y^2} = \sin(y) \).

VIII. Calculate each of the following limits:

1. \( \lim_{x \to \infty} \frac{x}{\sqrt{x^2 + x}} \)

2. \( \lim_{x \to -\infty} \frac{x}{\sqrt{x^2 + x}} \)

3. \( \lim_{x \to 0} \frac{\sin(7x)}{x} \)
IX. Find each of the following:

1. The most general form for an antiderivative of \(\csc^2(x) + 8x^3 - \sqrt{x}\) on the interval \(0 < x < \pi\).

2. \(f(x)\), if \(f'(x) = 6x^2 - 2\) and \(f(\pi) = 2\pi^3\).

3. The most general form for \(f(x)\) if \(f''(x) = 12x - 4\).

4. The differential of the function \(\sec^2(2x)\).

X. Let \(f: \mathbb{R} \to \mathbb{R}\) (i.e. let \(f\) be a function from the real numbers to the real numbers). In the blank to the left of each of the following questions, write the letter of the best response.

1. ______ What type of mathematical object is the graph of \(f\)?
   A) set  B) function  C) equation  D) codomain  E) number  F) operation

2. ______ If it exists, what type of mathematical object is \(\lim_{x \to 2} f(x)\)?
   A) set  B) function  C) equation  D) codomain  E) number  F) operation

3. ______ If it exists, what type of mathematical object is \(\lim_{x \to \infty} f(x)\)?
   A) set  B) function  C) equation  D) codomain  E) number  F) operation

4. ______ If it exists, what type of mathematical object is an antiderivative of \(f\)?
   A) set  B) function  C) equation  D) codomain  E) number  F) operation

5. ______ What type of mathematical object is \(f \circ f\)?
   A) set  B) function  C) equation  D) codomain  E) number  F) operation

6. ______ What type of mathematical object is composition of functions?
   A) set  B) function  C) equation  D) codomain  E) number  F) operation
XI. The table to the right shows the values of the functions \( f, g, f', \) and \( g' \) at the \( x \)-values 1, 2, 3, and 4. For example, \( f(4) = 2 \) and \( f'(4) = 4 \). Write the value of each of the following:

\[
\begin{array}{cccc}
  x & f(x) & f'(x) & g(x) & g'(x) \\
  1 & 2 & 3 & 1 & 4 \\
  2 & 3 & 2 & 4 & 1 \\
  3 & 4 & 1 & 4 & 2 \\
  4 & 2 & 4 & 1 & 2 \\
\end{array}
\]

\((g \cdot f)'(3) = \quad (g \circ f)'(1) = \quad (f/g)'(3) = \quad (g \circ g)'(4) = \quad\)

XII. Let \( g : \mathbb{R} \to \mathbb{R} \) be a function, not necessarily differentiable or continuous. Label each of the following statements either T for true or F for false.

_____ If \( \lim_{x \to a} g(x) \) exists, then \( g \) must be continuous at \( a \).

_____ If \( g \) is continuous at \( a \), then \( \lim_{x \to a} g(x) \) must exist.

_____ The Extreme Value Theorem applies only to differentiable functions.

_____ If \( g \) is continuous at \( a \), then it must be differentiable at \( a \).

_____ If \( g \) is differentiable at \( a \), then it must be continuous at \( a \).

_____ If \( f \) is continuous on a closed interval \( [a,b] \), and \( N \) is any value between \( a \) and \( b \), then there must exist \( c \) between \( f(a) \) and \( f(b) \) such that \( f(c) = N \).

_____ Both of the functions \( \cos^2(x) \) and \( \cos^2(x) + 5 \) are antiderivatives of \( -2 \sin(x) \).

_____ Every function of the form \( \frac{1}{x} + C \) for some value of \( C \) is an antiderivative of the function \( \frac{-1}{x^2} \).

_____ Every antiderivative of the function \( \frac{-1}{x^2} \) is of the form \( \frac{1}{x} + C \) for some value of \( C \).

_____ Newton’s method can be applied to find approximations to solutions to equations, as well as to find approximations to roots.

_____ If \( g \) is differentiable and \( G \) is an antiderivative of \( g \), then \( g' \) tells the concavity of \( G \).

_____ If \( g \) is differentiable and \( G \) is an antiderivative of \( g \), then \( g' \) tells where \( G \) is increasing or decreasing.