Mathematics 2443-003 Name (please print) ____________________

Examination II Form B

October 18, 2007

Instructions: Give brief answers, but clearly indicate your reasoning.

I. (5) Evaluate by changing to polar coordinates: $\iint_R (x+y)\,dA$, where R is the region that lies below the x-axis and between the circles $x^2+y^2=3$ and $x^2+y^2=4$.

II. (9) Let E be the upper hemisphere of the unit ball, that is, $E=\{(x,y,z) \mid x^2+y^2+z^2 \le 1,\ z \ge 0\}$. For the integral $\iiint_E f(x,y,z)\ dV$, supply the explicit limits of integration, the expression for dV, and (if necessary) the expressions for x, y, and z, that would be needed to calculate the integral:

(i) In xyz-coordinates (x,y,z)

(ii) In cylindrical coordinates (r,θ,z)

(iii) In spherical coordinates (ρ,θ,ϕ)

III. (5) Evaluate the integral $\iint_R e^{y^2}\,dA$, where $R=\{(x,y) \mid 0 \le y \le 1,\ 0 \le x \le y\}$.

IV. (5) Let E be the solid in the first octant bounded by $x^2+y^2+z^2=1$ and the three coordinate planes (that is, E is the portion of the unit ball that lies in the first octant). Suppose that the density at each point of E equals the distance from the point to the xz-plane. Write integrals to find the mass of E and its moment with respect to the xz-plane. Do not supply explicit limits for the integrals, or try to evaluate the integrals.

V. (5) Calculate the numerical value of a Riemann sum to estimate the value of $\iint_R x^2y\,dA$, where R is the rectangle $[0,4]\times[0,2]$, i. e. the (x,y) with $0 \le x \le 4$ and $0 \le y \le 2$. Partition the x-interval $[0,4]$ into two equal subintervals, and partition the y-interval into two equal subintervals, so that the Riemann sum has four terms. Use the Midpoint Rule to choose the sample points.

VI. (5) Sketch a portion of a typical graph $z=f(x,y)$, showing the tangent plane at a point $(x_0,y_0,f(x_0,y_0))$. Let $\vec{v}_x$ be the vector in the tangent plane whose $\vec{\imath}$-component is 1 and whose $\vec{\jmath}$-component is 0 (i. e. $\vec{v}_x$ is a vector of the form $\vec{\imath}+\lambda\vec{k}$ for some number λ). Show $\vec{v}_x$ in your sketch, and express λ in terms of f or its partial derivatives.

VII. (5) Calculate $\|\ (\vec{\imath}+f_x(x_0,y_0)\vec{k})\times(\vec{\jmath}+f_y(x_0,y_0)\vec{k})\ \|$. Give the details of the calculation, not just the answer.

VIII. (6) Find the surface area of the portion of the paraboloid $z=x^2+y^2$ that lies above the unit disk in the xy-plane.

IX. (5) Let E be the solid tetrahedron bounded by the coordinate planes and the plane $x+y+2z=2$. Supply limits for the integral $\iiint_E f(x,y,z)\,dV$, assuming that the order of integration is first with respect to x, then with respect to y, then with respect to z.

X. (5) Sketch the region and change the order of integration for $\int_1^3 \int_0^{\ln(x)} f(x,y)\,dy\,dx$.

XI. (5) Evaluate the integral $\int_{-a}^{a} \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)^{3/2}\,dx\,dy$.