Mathematics 2443-003 Name (please print) ____________________

Examination I Form B

September 18, 2007

Instructions: Give brief answers, but clearly indicate your reasoning. All functions are assumed to have continuous derivatives of all orders, so results such as Clairaut's Theorem may be freely applied if needed.

I. (7) For the function $f(x, y) = y^x$, tell the maximum rate of change of this function at the point $(2, 2)$. Find the rate of change of f at the point $(2, 2)$ in the direction toward $(-1, 3)$.

II. (5) Find the domain of the function $f(x, y, z) = e^{\sqrt{z-x^2-y^2}}$. Find its range (that is, the possible values that $f(x, y, z)$ assumes, as one considers all the points (x, y, z) in the domain of f). For finding the range, it may be useful to examine the values of f on the portion of the domain that lies on the z-axis.

III. (6) Sketch a portion of a typical graph $z = f(x, y)$, showing the tangent plane at a point $(x_0, y_0, f(x_0, y_0))$. Let $\vec{v}_x$ be the vector in the tangent plane whose $\vec{\imath}$-component is 1 and whose $\vec{\jmath}$-component is 0 (i. e. $\vec{v}_x$ is a vector of the form $\vec{\imath} + \lambda\vec{k}$ for some number λ). Show $\vec{v}_x$ in your sketch, and express λ in terms of f or its partial derivatives.

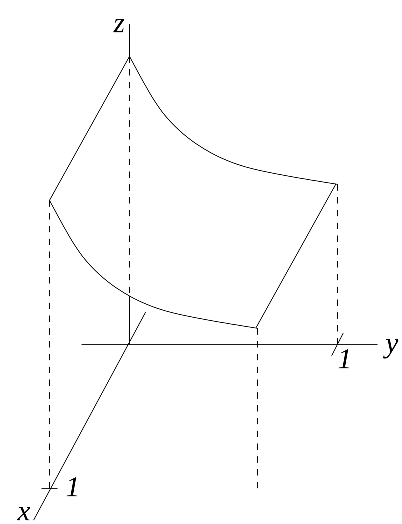
IV. (5) Calculate the differential $d(x^2 + y^2 + z^2)$. Use it to estimate $(1.1)^2 + 1 + (1.1)^2$ by calculating the linear part of the change of $x^2 + y^2 + z^2$ starting from the point $(1, 1, 1)$.

V. (5) Tell what Clairaut's Theorem says. Use Clairaut's Theorem to tell why there is no function $f(x, y)$ for which $\dfrac{\partial f}{\partial x} = \sin(xy)$ and $\dfrac{\partial f}{\partial y} = \cos(xy)$.

VI. (5) Use the Chain Rule to find $\dfrac{\partial R}{\partial x}$ when $x = 1$ and $y = 2$ if $R(u, v, w) = \ln(u^2 + v^2 + w^2)$, $u = x + 2y$, $v = 2x - y$, and $w = 2xy$.

VII. (3) A function R of the variables R_1, R_2, and R_3 is given implicitly by $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$. Use *implicit differentiation* to find $\dfrac{\partial R}{\partial R_1}$, $\dfrac{\partial R}{\partial R_2}$, and $\dfrac{\partial R}{\partial R_3}$.

VIII. (4) In an xy-coordinate system, sketch the gradient of the function whose graph is shown to the right.



IX. (4) Find all critical points of the function $f(x, y) = x^4 + y^4 - 4xy + 2$.

X. Let T be the triangle bounded by the x-axis, the y-axis, and the line $x + y = 1$. Find the maximum and
(5) minimum values of $f(x, y) = 2x^2 + y^2$ on:

(a) The bottom side of T, i. e. the side that lies in the x-axis.

(b) The diagonal side of T, i. e. the side that lies in the line $x + y = 1$.

XI. Use Lagrange multipliers to find the extreme value or values of $f(x, y) = 2x^2 + y^2$ on the line $x + y = 1$.
(5)