I. Evaluate by changing to polar coordinates: \( \iint_R x + y \, dR \) where \( R \) is the region between \( x^2 + y^2 = 1 \) and \( x^2 + y^2 = 2 \) and above the \( x \)-axis.

II. For the function \( f(x, y) = \ln(x^2 + y^2) \), find the maximum rate of change at the point \((1, 2)\), and the direction in which it occurs. Find the directional derivative of \( f \) at \((1, 2)\) in the direction toward \((2, 4)\).

III. Let \( S \) be the portion of the sphere of radius \( a \) that lies in the first octant. Use the standard parameterization of \( S \) to calculate \( \iiint_S (y \vec{i} - x \vec{j} + \vec{k}) \cdot d\vec{S} \).

IV. Use the Divergence Theorem to calculate the surface integral \( \iiint_S (x^2 z^3 \vec{i} + 2xyz^3 \vec{j} + xz^4 \vec{k}) \cdot d\vec{S} \), where \( S \) is the surface of the box with \( 0 \leq x \leq 3 \), \( 0 \leq y \leq 2 \), \( 0 \leq z \leq 1 \).

V. The radius of a right circular cone is increasing at a rate of 6 in/s while its height is decreasing at a rate of 3 in/s. At what rate is the volume \( V = \pi r^2 h/3 \) changing when the radius is 10 and the height is 5?

VI. Let \( S \) be the upper half of the sphere of radius 2, that is, the points \((x, y, z)\) with \( x^2 + y^2 + z^2 = 4 \) and \( z \geq 0 \), and suppose that \( S \) is oriented with the upward normal. Use Stokes' Theorem to evaluate \( \iint_S \text{curl}(x^2e^{yz} \vec{i} + y^2e^{xz} \vec{j} + z^2e^{xy} \vec{k}) \cdot d\vec{S} \).

VII. Let \( S \) be the upper half of the sphere of radius 1, that is, the points \((x, y, z)\) with \( x^2 + y^2 + z^2 = 1 \) and \( z \geq 0 \). Using the geometric interpretation of the surface integral of a vector field as the “flux” (that is, not by calculation using a parameterization or a formula from the formulas list), explain each of the following equalities:

1. \( \iint_S \vec{F} \cdot d\vec{S} = 0 \)
2. \( \iint_S \vec{k} \cdot d\vec{S} = \pi \)

VIII. Verify that the function \( u = \cos(x - at) + \ln(x + at) \) is a solution to the wave equation \( u_{tt} = a^2 u_{xx} \).

IX. Let \( S \) be the portion of the cylinder \( x^2 + z^2 = 4 \) that lies between the vertical planes \( y = 0 \) and \( y = 2 - x \).

1. Calculate \( \vec{r}_\theta, \vec{r}_h, \vec{r}_h \times \vec{r}_\theta \), and \( \| \vec{r}_h \times \vec{r}_\theta \| \).
2. Calculate \( \iint_S x \, dS \).
3. Calculate \( \iint_S x \vec{k} \cdot d\vec{S} \).

X. The curl of the vector field \( y \vec{i} - z \vec{j} + x \vec{k} \) is \( \vec{i} - \vec{j} - \vec{k} \). Let \( S \) be the triangle which is the part of the plane \( 2x + y + z = 2 \) that lies in the first octant. Give \( S \) the upward normal, and give its boundary \( C \) the corresponding positive orientation. Use Stokes’ Theorem to evaluate the line integral \( \int_C (y \vec{i} - z \vec{j} + x \vec{k}) \cdot d\vec{r} \).

(Hint: the surface integral on \( S \) is easy to calculate if one uses the definition \( \iint_S \vec{G} \cdot d\vec{S} = \iint_S \vec{G} \cdot \vec{n} \, dS \).)
XI. In an $xy$-coordinate system, sketch the gradient of the function whose graph is shown to the right.

XII. A function $R$ of the variables $R_1$, $R_2$, and $R_3$ is given implicitly by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. Use implicit differentiation to find $\frac{\partial R}{\partial R_3}$.

XIII. Use the Divergence Theorem to show that if $E$ is a solid with boundary the surface $S$, then $\int \int_S \left( \frac{x}{3} \mathbf{i} + \frac{y}{3} \mathbf{j} + \frac{z}{3} \mathbf{k} \right) \cdot d\mathbf{S}$ always equals the volume of $E$.

XIV. On two different coordinate systems, graph the following vector fields:

1. $\vec{F}(x, y) = x \mathbf{i} + y \mathbf{j}$
2. $\vec{F}(x, y) = \frac{-y}{x^2 + y^2} \mathbf{i} + \frac{x}{x^2 + y^2} \mathbf{j}$

XV. Sketch the region and change the order of integration for $\int_0^1 \int_{e^x}^e f(x, y) \, dy \, dx$. 